Gender pay gap

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Abstract: The article explains the origins of the gender pay gap as a Nash equilibrium of a game with incomplete information about women's work at home and in the marketplace based on the earlier results of Chichilnisky (2005, 2008a, 2008b). Expectations about women's lower market wages leads to the over utilisation of women in the household, and this, in turn, leads to lower productivity and lower wages for women in the marketplace. The situation is rational, but (as the prisoner's dilemma) it is generally Pareto inferior. Inequity at home breeds inequity in the marketplace and reciprocally, leading to a persistent gender gap. With learning by doing, at high levels of skill there is a Pareto superior equilibrium where men and women share efforts equally at home and receive the same pay in the marketplace, firms enhance their profits, and there is additionally more welfare at home. Updated Family Law and appropriate contracts can help resolve this Pareto inferior situation as well as increase productivity and economic growth in the economy as a whole (Pyle, 1990).

Keywords: family and the market; women in green economics; vicious circle of discrimination; economic losses from gender bias; multiple equilibrium; Nash equilibrium; division of labour; specialisation and efficiency; gender gap; learning by doing; family law.

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Biographical notes: Graciela Chichilnisky has worked extensively in the Kyoto Protocol Process, creating and designing the carbon market that became international law in 2005. She also acted as a Lead Author of the Intergovernmental Panel of Climate Change, which received the 2007 Nobel Peace Prize. She has created the concept of basic needs in economic development and introduced the formal theory of sustainable development, is a frequent Key Note Speaker around the world, a Special Adviser to UN organisations and heads of state. Her pioneering work uses market mechanisms to reduce carbon emissions, conserve biodiversity and ecosystem services, and reducing the global wealth divide. The UNESCO Professor of Mathematics and Economics 1996–2008, she taught previously at Harvard and Stanford Universities, is a UNESCO Professor of Economics and Mathematical Statistics at Columbia University in New York, USA (1996–2009), she is also the Sir Louis Matheson Distinguished Professor at Monash University, Australia. Her website is www.chichilnisky.com.

1 Introduction

Green economics is a shift in economic values towards the future and the global commons (Chichilnisky, 1994). It embodies a concern for the survival of the human species. Green economics has therefore a special role for women (Kennet, 2009). Women's unique contribution to reproduction and family welfare is critical to the survival of human societies. In much of the developing world that makes up 80% of the world's population, women replace other forms of energy as well as animals in providing water and wood for kindling, making food and clothing to nurture and shelter their families against hunger and the elements. The great paradox of our times is that in the world as whole women are undervalued and overworked, underpaid and degraded in their living and working conditions (Chichilnisky, 2005, 2008a, 2008b). This irrational situation parallels other short sighted economic values that have landed us into major global environmental dilemmas, catastrophic global warming risks and biodiversity extinction. The degradation of women and their value transcends cultures and economic systems in the modern era, and even in the centres of higher learning in leading economies such as the US' most distinguished universities, we find economists describing women in public and official events as 'genetically inferior to men'.[1] The current situation has evolved over time. Women had lower salaries historically, and performed most housework because men could make a higher income working in the marketplace. Under the prevailing conditions, this division of labour is a rational way to maximise family income. However, the burden of excessive housework decreases the time and the energy that women can bring to the marketplace. The family produces *externalities* on the firm.[2] In such a situation, women appear to be more *risky* because they may not be available in case of emergencies; for example, if they act as the main providers of the medical needs of the family.[3] If workers are assets, then women appear to be riskier assets even when they are equally productive. This perceived riskiness is, in turn, used to justify women's lower wages, closing the vicious circle. From this game between the family and the marketplace, the gender gap emerges as a rational and stable Nash equilibrium. It is a rational but yet undesirable situation for the players, similar to the classical prisoner's dilemma.

Women's riskiness is felt most acutely in the most demanding and highest paid jobs where 24 × 7 availability is often required – creating a larger gap in women participation and salaries at the highest levels and explaining the glass ceiling (Meyersson Milgrom et al., 2001; Meyersson Milgrom and Petersen, 2003; Chichilnisky, 2005, 2008a, 2008b).

The empirical and experimental evidence appears to confirm these observations (Rosholm and Smith, 1996; Gupta Oaxaca and Smith, 2003; Bureau of Labor Statistics, 2003; Blau and Kahn, 2004; Gunther, 2004; NCES, 2004; Cahuc and Zylberger, 2004; Blackaby Booth and Frank, 2005; Chichilnisky and Fredriksen, 2008). Recent empirical research in 28 nations confirms that women work at home 70% more than men (Davis et al., 2007), and the specific results in this article have been tested empirically with US household data (Schachmarove, 2009). Additionally, Bonke et al. (2005) found that larger differences between men and women's work at home are associated with larger differences in market salaries. Recent experiments by Gneezy et al. (2003) show that women perform worse than men in competitive environments. This makes sense. Due to their lower salaries, women spend more time working at home where the most important skills involve sharing and cooperation. Success in the marketplace requires instead more

competitive skills.[4] One can therefore expect women to adapt to the cooperative family 'mores', while men adapt instead to the competitive 'mores' of the marketplace.

This article formalises a toy game where women and men share their time between the family and a Walrasian market economy. They *learn by doing*, in the sense that the more they work the more productive they are. This follows the classic article by Becker (1985), which provides the standard argument for specialisation of women and men, at home or in the marketplace respectively.[5] In contrast with Becker's assumption, however, I follow the seminal article of Arrow (1962) where he introduced *learning by doing* with decreasing returns. This seems entirely appropriate after a certain number of hours per day. I show that Arrow's assumption reverses Becker's findings in the sense that specialisation is no longer necessary for efficiency. There is now another, rational, solution in which women and men are paid the same and share the work equally in both institutions. This fair outcome emerges at higher levels of output and skill, when the economy is richer and more productive. Once production exceeds a minimum level, we enter Arrow's regime where the learning curve is concave rather than convex as in Becker's work. I show that a new equilibrium emerges that leads to more welfare at home, more family services, and simultaneously to higher productivity and profits in the marketplace. Specialisation and inequity is no longer the only solution – now fairness is Pareto efficient.

If such equitable solutions exist, one may ask, why are not they used more often? The answer is simple: under our current economic and social conditions, the equitable solutions seems riskier and are avoided, as it happens in is the optimal solution in the well-known *prisoner's dilemma*. Equal treatment in the family depends on equal treatment in the marketplace and vice versa – but in the absence of cooperative rules neither institution can safely depend on the other. In the conclusions, I suggest updating Family Law to create equal treatment for both genders in inheritance, children's custody, retirement benefits, and the right to travel and work outside the house without a spouse's permission. In addition, improved contractual arrangements between the family and the marketplace can help reach efficient and equitable social solutions and Equal Pay Act should become law (it is not) in the USA. Empirical work (Pyle, 1990) has documented that updating Family Law can increase economic growth and benefit an entire nation.

2 The firm

The model was introduced in Chichilnisky (2008a, 2008b). Since firms are competitive, they take the price of good x, p_x and wages of men and women w_1 and w_2 as parametrically given. Maximising profits implies the standard condition that wages must equal the marginal product of labour:

$$w_1 = \frac{\partial f}{\partial L_1} \quad \text{and} \quad w_2 = \frac{\partial f}{\partial L_2} \tag{1}$$

There are two *externality* parameters γ_1 and γ_2 which reflect how a person's work at home and influence their productivity in the marketplace. These are *externalities* that the home inflicts on workplace. The firm takes these parameters as given as it has no way to

find out how much time a person works at home, it is private information; and so they represent an 'externality' to the firm:

$$x = f(L_1, \gamma_1) + f(L_2, \gamma_2)$$

Since the firm cannot observe nor gather information (in the US, by law) about the extent of women's household work, for each given γ_1, γ_2 profit maximisation by the firm implies that (1) should be rewritten as:

$$w_1 = \frac{\partial f}{\partial L_1}(\gamma_1) \quad \text{and} \quad w_2 = \frac{\partial f}{\partial L_2}(\gamma_2)$$

3 The family

There are several identical families. Neglecting distributional issues within the family, a family's welfare derives from family services h, and from the consumption of good x. The family goal is to optimise welfare:

$$Max(U(x,h)) \tag{2}$$

Family services are produced according to a technology g

$$h = g(l_1) + g(l_2) \tag{3}$$

where l_1 and l_2 are two types of labour in the household, men's and women's respectively. Let K be the total amount of hours that a person can feasibly work in a given period of time, at home and in the market. As an example, in a given day, this could be $K = 15$. When all labour is utilised:

$$L_1 = K - l_1 \quad \text{and} \quad L_2 = K - l_2 \tag{4}$$

The family's *income* equals the wages that its men and women earn in the marketplace plus the firms' profits, since families own the firms. The value of what the family buys $p_x x$ must equal its income:

$$p_x x = w_1 L_1 + w_2 L_2 + \pi \tag{5}$$

where profits π are the firm's revenues minus its costs:

$$\pi = p_x\left(f(L_1, \gamma_1) + f(L_2, \gamma_2)\right) - (w_1 L_1 + w_2 L_2) \tag{6}$$

We normalise by assuming that the price of x is one, $p_x = 1$, so that the family's 'budget' equation is:

$$x = (5) + (6) = f(L_1) + f(L_2) \tag{7}$$

3.1 The family's trade-off

The family faces a trade-off in deciding whether to use labour at home or in the marketplace. The more labour is used at home, the more family services are produced, but the lower is the family's income and therefore the fewer market goods it consumes. The family allocates labour at home and in the marketplace to optimise its welfare.

When women and men are paid differently, $w_1 \neq w_2$, the family's decision problem by (1) to (7) is to choose l_1, l_2 to:

$$Max_{l_1,l_2} U\left(f\left(K-l_1,\gamma\right)+f\left(K-l_2,\gamma\right), g\left(l_1\right)+g\left(l_2\right)\right) \tag{8}$$

From (1), this implies:

$$\frac{\partial U}{\partial x}\left(-w_1\right)+\frac{\partial U}{\partial h}\frac{\partial g}{\partial l_1}=0 \tag{9}$$

and

$$\frac{\partial U}{\partial x}\left(-w_2\right)+\frac{\partial U}{\partial h}\frac{\partial g}{\partial l_2}=0$$

This links wages and the productivity of each type of labour at home and, furthermore, wages determine the amount of time each works at home:

$$\frac{\partial g}{\partial l_1}=\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}}w_1 \quad \text{or} \quad w_1=\frac{\partial g}{\partial l_1}\frac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial x}} \tag{10}$$

$$\frac{\partial g}{\partial l_2}=\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}}w_2 \quad \text{or} \quad w_2=\frac{\partial g}{\partial l_2}\frac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial x}} \tag{11}$$

3.2 Public goods and common property resources

Acting as a single unit, the family makes choices about how to allocate women and men's labour, namely l_1 and l_2. This means that labour is treated as *common property* by the family. Furthermore, since we consider a single welfare level for the entire family, this means that family services are shared as a 'public good' within the family. See also Apps and Rees (1997) and Aronsson et al. (2001).

In summary: *the family produces a public good using common property resources*. Family services are better described as a 'local' public good within the family, because they not shared with other families.

4 Learning by doing: empirical evidence of logistic curves

Arrow (1962) pointed out that the more time we spend in a given activity the better we become at doing it. This is called *learning by doing*. Becker (1985) assumed that marginal productivity $\dot{g}$ increases with time. Under these conditions, each person in the family (man or woman) should specialise – one should specialise in working at home, and the other in the marketplace. Becker's model implies that this way both are more productive, at home and in the marketplace, thus increasing family welfare. As a direct consequence of Becker's assumption, when women's salaries are lower than men's, women should do all the housework. Men should only work in the marketplace.

Since women's salaries are lower than men's historically and currently, Becker's assumption leads directly to a division of labour where women stay at home and men work in the marketplace. Under Becker's assumptions, traditional division of labour is a rational and efficient solution.

There is indeed learning by doing in our society and therefore, Becker's assumption is realistic, but only up to a point. Human beings need rest after a number of working hours, and this implies a decrease in productivity beyond a certain number of hours of work. Accordingly, we assume here that the time derivative of the home production function $\dot{g}$, is initially positive but after a maximum is reached, $\dot{g}$ starts to decrease since humans cannot work productively without rest. If $g(t)$ is the amount of h produced with t hours worked, we may assume that increases in productivity follow a modified quadratic form, increasing initially and then decreasing as was just postulated,

$$\dot{g}_t = H\left(g_t\right) = \beta g - \lambda g^2 \quad \text{with} \quad \beta, \lambda > 0$$

This equation integrates to yield the classic logistic curve that is used often to describe the evolution of biological populations over time illustrated in the figure Chichilnisky (2008b):

$$g(t) = \frac{\beta g_0}{\lambda z_0 + \left(\beta - \lambda z_0\right)\exp\left(-\beta t\right)}$$

This curve is initially convex and after the inflection point $t = \frac{\beta}{\lambda}$ it becomes concave. The convex part is similar to Becker's assumption and yields similar results. On the other hand, the concave part, which occurs after the inflection point is reached yields very different results as is shown below. The inflection point determines a change from one regime to the other; it is as the maximum of the quadratic curve, which is the time derivative of g.

Assumption 1: In the following, we assume that production has reached the inflection point at home and at the marketplace, an assumption that seems to tally with the evidence. We describe this as having achieved *higher levels of output*.

4.1 Equity at home improves welfare

Proposition 1: At higher levels of average output, equity benefits the family.

Distributing home labour equally between men and women produces more household services for the same total labour. Formally, if

$$\frac{l_1 + l_2}{2} > \frac{\beta}{\lambda}$$

then

$$l_1 \neq l_2 \Rightarrow g(l_1) + g(l_2) < 2g\left(\frac{l_1 + l_2}{2}\right)$$

Proof: (see Chichilnisky, 2008a, 2008b)

$$2g\left(\frac{l_1 + l_2}{2}\right) > g(l_1) + g(l_2) \Leftrightarrow$$
$$g\left(\frac{l_1 + l_2}{2}\right) > \frac{g(l_1)}{2} + \frac{g(l_1)}{2},$$

which is the definition of concavity. Above its inflection point, the logistic curve g is concave since its second derivative is negative, proving the inequality. Equity is a more efficient use of resources at home whenever:

$$\frac{l_1 + l_2}{2} > \frac{\beta}{\lambda},$$

as we wished to prove.

Formally, equity is a more efficient use of resources at home after the inflection point.

4.2 Inequity at work leads to inequity at home

There is historic difference in the average pay of men and women, about 25% or 30% in the US. What is the optimal response by the family to this inequity, in terms of allocating labour at home? The following proposition provides a response:

Proposition 2: Inequity at work leads to inequity at home

Proof: (see Chichilnisky, 2008a, 2008b)

When women are paid less than men in the marketplace, $w_1 > w_2$, the family's optimal response is that women should work longer hours at home than men. When the gender difference in wages is large enough, $\frac{w_1}{w_2} > M = \frac{\sup \frac{\partial g}{\partial l_1}}{\inf \frac{\partial g}{\partial l_2}}$ it is optimal for the family that women should do all the housework, and men should work only in the marketplace.

Furthermore, women work at home up to the point where their marginal productivity is lower than men's. As we saw in Section 4, when $g(t) > \frac{\beta}{\lambda}$, the marginal productivity

of labour $\frac{\partial g}{\partial l_2}$ is a decreasing function of the time allocated, so that lower productivity means longer hours for women at home.

Finally, when the ratio of salaries exceeds M, the ratio of the supremum and the infimum productivity of g, namely when:

$$\frac{w_1}{w_2} > M = \frac{\sup \frac{\partial g}{\partial l_1}}{\inf \frac{\partial g}{\partial l_2}} \tag{12}$$

it is optimal that women should completely specialise in housework.

Proposition 2 implies that it is always optimal for the family to use more women labour at home when they have lower salaries than men. If women's housework hours are less than the maximum feasible, K, then it would be rational that women should also work in the marketplace in addition to their work at home – at reduced salaries. Furthermore, when salary differentials are large enough, it is optimal for the family that women do all the housework and that they work also in the marketplace at reduced salaries while men, on the other hand, work only in the marketplace and at higher salaries. Women are overworked and underpaid.

The logic of the situation (Chichilnisky, 2008a, 2008b; equation (13)] implies that when $w_1 > w_2$, then women's marginal productivity is lower than men's at home and also in the marketplace. When production functions f and g are concave, this implies in turn that women work more hours than men at home and also in the marketplace, because marginal productivity decreases with the time worked:

$$L_1 > L_2 \quad \text{and} \quad l_1 > l_2 \tag{13}$$

However, observe that

$$L_1 = K - l_1 \quad \text{and} \quad L_2 = K - l_2$$

implies

$$L_1 > L_2 \Rightarrow l_2 > l_1 \tag{14}$$

How to reconcile (13) and (14)? The former would imply the opposite inequality than the latter. However, when externalities are taken into account, the explanation is simple. In the next section, we show that the externality that the home produces on the firm, namely the function γ_1 and γ_2, reconcile these two apparently divergent inequalities.

4.3 Externalities: inequity at home reduces women's productivity in the market

As already pointed out, the amount of work that a person performs at home has an impact on their productivity in the marketplace. The first hour that a woman works at the firm may be the sixth hour of work that day, since she may have worked already five hours at home. The more she works at home, the less productive she is in the marketplace.

Yet, the numbers of hours that a person works at home are not known to the firm, nor can the firm control them. This is an *externality* that the family causes the firm. Formally,

l_1 and l_2 are treated as parameters by the firm even though they have an impact on the firm through worker's productivity. These observations may be formalised as follows:

Assumption 2: There exists a single parameter $\gamma > 0$ representing an 'externality' on the firm so that for $i = 1, 2$

$$\text{For } i = 1,2 \qquad \frac{\partial f}{\partial L_i} = \frac{\partial f}{\partial L_i}(\gamma) \quad \text{where} \quad \frac{\partial^2 f}{\partial \gamma \partial L_i} < 0.$$

A simple example of this phenomenon would be

$$f(L_i) = \gamma(l_i) L_i^{\alpha}$$

where

$$\gamma = \gamma(l_i) \quad \text{and} \quad \partial\gamma / \partial l_i < 0.$$

Under Assumption 2 above:

Proposition 3: Inequity at home leads to lower productivity of women at work, and to lower salaries for women.

Proof: This is a consequence of Assumption 2, and equations (10) and (11) above, see Chichilnisky [2008b; equation (13)].

The productivity of women in the marketplace depends on the amount of time they work at home. This breaks the symmetry between productivity at work and hours worked. Even if the production function f is concave, those who spend more time working at home could have lower productivity in the marketplace while working fewer hours than the rest. This is because the production function f depends not only on L but also on l and at higher levels of l, the graph of $f(L)$ shifts downwards due to the externality. This resolves the apparent conflict in (13) and (14) above.

4.4 Inequity lowers family welfare

We saw that inequity at work leads to inequity at home and reciprocally inequity at home reduces productivity at work for those working longer hours at home. If women are subject to this inequity, then obviously they are worse off under these conditions. Is it possible, however, that the family as a whole is better off? The following proposition provides a response.

Proposition 4: At higher levels of output, inequity lowers family welfare, decreasing both family services h and the family's consumption of market goods x.

Proof: (see Chichilnisky, 2008b)

We have already shown that, under the conditions, the family produces more home services h with the same total amount of labour if the work load is distributed equally between the two genders. Namely, when $\frac{l_1 + l_2}{2} > \frac{\beta}{\lambda}$

$$l_1 \neq l_2 \cdot \Leftrightarrow 2g\left(\frac{l_1 + l_2}{2}\right) > g(l_1) + g(l_2)$$

Therefore, inequity leads to less family services h. Yet, it is still possible that inequity at home could increase family income sufficiently to compensate for the loss in family services. We show that this is not possible under the conditions. By definition, inequity at home means $l_1 < l_2$

which implies $L_1 = K - l_1 > L_2 = K - l_2$

This under the conditions implies that women's marginal productivity at work is lower than men's, see Chichilnisky [2008b; equation (13)]. Since the firm has a logistic production function f then for the same total amount of labour $L_1 + L_2$ an equal workload among women and men increases total output:

$$2f\left(\frac{L_1 + L_2}{2}\right) > f(L_1) + f(L_2) \quad \text{when} \quad L_1 \neq L_2$$

as shown in Proposition 1 above. Therefore in the unequal situation, the total production of market goods x is lower than when men and women share work equally. Since all production is consumed by families, the family consumes less market goods x as well as fewer family services. Therefore, inequity at home lowers the family's welfare.

4.5 Inequity leads to lower output and lower profits

Proposition 5: At higher output levels, inequity reduces the firm's output and lowers its profits.

Proof: (Chichilnisky, 2008a, 2008b) We saw in Proposition 4 that under the conditions, inequities decrease the market's output of x. For the same total amount of work, the production of the firms is higher when men and women divide equally the work load:

$$2f\left(\frac{L_1 + L_2}{2}\right) > f(L_1) + f(L_2) \quad \text{when} \quad L_1 \neq L_2,$$

it remains to consider the impact of inequity on profits, namely on the function

$$\pi(L_1, L_2) = f(L_1) + f(L_2) - w_1 L_1 - w_2 L_2$$

We wish to compare

$$\pi(L_1) + \pi(L_2) \quad \text{with} \quad 2\pi\left(\frac{L_1 + L_2}{2}\right)$$

By concavity (since we are above the inflection point of f) profits increase with the level of output, namely

$$\frac{\partial \pi}{\partial x} > 0$$

Since equity increases output, and profit is an increasing function of output, it follows that equity increases profits as well. Equivalently, inequity decreases output and profits.

4.6 A Nash-Walrasian solution

This section describes the functioning of the economy as a whole (Chichilnisky, 2008a, 2008b). The economy consists of a Walrasian market where firms maximises profits, and of families that produce public goods using common property resources, maximising welfare. There are three traded goods in the economy: the market good x, women's labour, and men's labour. We normalised the price of x so that $p_x = 1$.

Recall that the family is not Walrasian; its services h are shared among the members, which make them similar to (local) public goods. Furthermore, the resources such as labour l_1 and l_2 that are used to produce h are allocated by common decision within the family so as to maximise the family's welfare. Therefore, the family treats resources as common property. Additionally, the family produces an externality on the firm γ which depends on the hours that men and women work at home, $\gamma = \gamma(l_i)$, $i = 1, 2$. There are no benchmark models to analyse the functioning of such a mixed economy. We need some definitions.

Definition: If $w_1 \neq w_2$ we say that the *market is unfair*. If $w_1 = w_2$ we say that the *market is fair*.

Definition: If $l_1 \neq l_2$ we say that *the family is unfair* and if $l_1 = l_2$ we say that *the family is fair*.

Proposition 6: Given wages for the two types of labour, w_1 and w_2, from the family's welfare optimisation behaviour it is possible to determine

1 the amount of family services it produces

2 the employment of men's and women's labour at home, l_1 and l_2

3 the offer of labour of the two types to the marketplace, $K - l_1$ and $K - l_2$

4 the family's demand for market goods

5 the family's income

6 its welfare level

7 the value of the externality parameters which modify the firm's production function.

On the other hand, the firm has expected values for the productivity parameters and from the firm's profit maximisation behaviour it is possible to determine

8 the amount of labour the firm wishes to employ (men and women)

9 how much it produces

10 what are its profits

11 the productivity of its labour.

Proof: This is a standard microeconomic exercise, see Chichilnisky (2008a, 2008b). In Proposition 6, the family and the firm may have contradictory goals in terms of the productivity parameters γ_1^e and γ_2^e, the market goods produced and consumed and people employed. A solution for this economy arises when firms and families behave consistently:

Definition 1: A Nash-Walrasian solution for this economy consists of wages for men and for women w_1^*, w_2^* and expected values of the externality parameters $\gamma_1^e \gamma_2^e$ leading to consistent behaviour by the family and the firm: the levels of employment and consumption that derive from profit optimisation by the firm and from welfare optimisation by the family, clear all three markets, and the value of the externality produced by the family on the firm equal the values expected by the firm.

In particular, a solution satisfies:

(0) expectations are confirmed

$$\gamma(l_1) = \gamma_1^e \quad \text{and} \quad \gamma(l_2) = \gamma_2^e$$

(1) supply of men's labour equals demand for men's labour by the firm

$$L_1^D(w_1, w_2) = N.\arg\max \pi(w_1, w_2) = L_1^S(w_1, w_2) = 15 - l_1(w_1, w_2) \tag{15}$$

(2) supply of female labour equals demand of women's labour by the firm

$$L_2^D(w_1, w_2) = N.\arg\max \pi(w_1, w_2) = L_2^S(w_1, w_2) = 15 - l_2(w_1, w_2) \tag{16}$$

and

(3) supply by the firm of x equals the family's demand for x,

$$x^S(w_1, w_2) = f\left(L_1^D(w_1, w_2), L_2^D(w_1, w_2)\right) = x^D(w_1, w_2) = w_1 L_1 + w_2 L_2 + \pi \tag{17}$$

The existence of a solution shows that the model as postulated is internally consistent.

Proposition 7: There exists a solution for this economy.

Proof: In the Appendix.

5 The market – family game

This section defines a game with two players, the market and the family. The *market's objective* is to maximise profits as defined in (1). The *family's objective* is to maximise welfare as defined in (2). The players choose their strategies to achieve their goals. The market's strategy is to set wages for men and for women, w_1 and w_2, and expectations about their productivity γ_1^e and γ_2^e while the *family's strategy* is to allocate labour at home among men and women, l_1 and l_2.

Definition: A Nash equilibrium is a set of strategies for the market and for the family $\left(w_1^*, w_2^*, \gamma_1^e, \gamma_2^e, l_1^*, l_2^*\right)$ leading to a solution for the economy in which each player reacts optimally to the other's strategy, so neither has an incentive to deviate.

Proposition 8. At high levels of output:

1 There is a Nash equilibrium where women have lower salaries. The family reacts by allocating more house work to women. Conversely, at a Nash equilibrium where the family allocates more housework to women, women's productivity is lower in the marketplace and they receive lower salaries than men. This Nash equilibrium is called *unfair-unfair*.

2 There is a Nash equilibrium where women have the same salaries as men. Women have the same productivity. The family reacts by sharing equally housework between men and women. Conversely, at a Nash equilibrium where women and men share housework equally, their wages in the marketplace are the same. This is a *fair-fair* Nash equilibrium.

3 The *unfair-unfair* Nash equilibrium is Pareto inferior. The *fair-fair* Nash equilibrium is Pareto efficient, but in a transition it appears to be riskier.

Proof: (see Chichilnisky, 2008b) When women have the same salaries as men, both bring the family the same income for the same hours in the marketplace. By Chichilnisky [2008b; equation (13)] their productivity is the same at an optimum, and given the assumptions, it is more productive for both men and women to work the same hours in the marketplace. At the same time, by Proposition 1, women work at home the same number of hours as men, since under the conditions, sharing work equally at home provides more family services for the same total amount of labour.

Reciprocally, when women and men share work equally at home, then it is optimal for the firm to pay both equally from Chichilnisky [2008b; equation (13)]. The *fair-fair* pair of strategies just described is a Nash equilibrium of the market-family game because when following such a pair of strategies, each player is responding optimally to the others' move.

At a Nash equilibrium, where women's salaries are inferior to men's, it is optimal for the family to choose an unfair distribution of household work by Proposition 2. Women work more at home, and their productivity at home is lower as shown in Proposition 2 and in Section 4.4, and so is their productivity at work by Chichilnisky [2008b; equation (13)]. This is an *unfair-unfair* Nash equilibrium, with both players responding optimally to each other. Nevertheless, it is a Pareto inferior solution.

The *first fair-fair* equilibrium is Pareto optimal. The following section illustrates why the fair-fair equilibrium appears to be riskier under the conditions. This completes the proof.

5.1 A matrix game

The matrix below illustrates a game where the horizontal strategies represent the market's and the vertical strategies represent the family's. The payoffs for the market are sub-indexed 1 and those for the family are sub-indexed 2.

$$\begin{pmatrix} & w_1 \neq w_2 & w_1 = w_2 \\ l_1 \neq l_2 & (A_1, A_2) & (C_1, D_2) \\ l_1 = l_2 & (D_1, C_2) & (B_1, B_2) \end{pmatrix}$$

In this matrix game, Proposition 8 can be summarised by the inequalities

$$C_1 < A_1 < B_1 < D_1$$

and

$$C_2 < A_2 < B_2 < D_2$$

when (A_1, A_2) is the outcome of the unfair-unfair Nash equilibrium, (B_1, B_2) is the outcome of the fair-fair Nash equilibrium. The *fair-fair* Nash equilibrium is Pareto efficient because $A_1 < B_1$ and $A_2 < B_2$.

The Pareto efficient Nash equilibrium appears to be more risky, because $C_1 < A_1$ so if the firm plays *fair* but the family plays *unfair* the market will be worse off, this is Proposition 3. Conversely, $C_2 < A_2$ implies that the family will be worse off if it plays *fair* while the market plays *unfair*, by Proposition 2.

5.2 The family-market game is similar to the prisoner's dilemma

The matrix game represented above has a similar structure to the 'prisoner's dilemma's game'.

6 Conclusions

The coupling of two distinct institutions – the market and the family – can lead to a disproportionate allocation of home responsibilities to women, and simultaneously to the lowering of women's wages. We showed that there is a Pareto optimal solution that is better for all, involving equity at home and in the workplace. However, this solution seems riskier because if either the family or the firm deviate from it, the other is worse off. The family loses if it plays fair when the market does not, and vice versa (Edin and Richardson, 2002; Elul et al., 2002; Engineer and Welling, 1999).

What social institutions can help resolve this problem? Waldfogel (1998) and others have considered similar issues.

A prenuptial agreement that specifies women's and men's roles in the family could be a start. It should have penalty attached if the parties default from what was promised. Using such a legal agreement, women can present themselves at work as fully able to deliver so a fair employer is not misled about the nature of the labour it hires, about the obligations she has at home and the externalities these produce.

Similarly, strengthening equal pay provisions in the marketplace should support the execution of these prenuptial agreements. This requires voting and enforcing the Equal Pay Act – and perhaps making this enforcement contingent on the availability of the

prenuptial agreement just discussed. This way the firms would not risk being penalised for playing fair.

Other solutions include updating Family Law, which is out of date in many nations. This solution has been shown to yield aggregate positive dividends for the economy as a whole, in terms of national productivity and economic growth in nations such as Ireland (Pyle, 1990). Such a solution would be available to the state, and yet would be able to improve conditions within the family. Therefore, it can address the family-market prisoner's dilemma pointed out here. Family Laws for this purpose include equal treatment in:

1 inheritance

2 children's custody

3 retirement benefits

4 freedom of working outside the home and to travel without spousal permission, among others.

In general terms, solutions to the prisoner's dilemma have been proposed over the years, most of them encourage cooperation among the players. Often, this requires repeated games among the players, which may not be realistic in the case of marriage (Lagerlöf, 2003). In any case, any solution that encourages a cooperative outcome between the family and the market will benefit both. The moral of this article is that equity may appear to be riskier – and indeed it may initially be – but it is after all the Pareto efficient allocation. Room should be made for the missing contracts between the players – the market and the family – and the introduction of updated Family Law that takes advantage of the existence of a win-win solution, making everyone better off.

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Notes

1 This refers to official statements made by Larry Summers, currently adviser to Pres. Obama, acting as President of Harvard University six years ago, statements that led to his eventual dismissal, as documented in *The New York Times* and widely debated in the US press and elsewhere.

2 In economic terms, there are *externalities* between the market and the family because the more a person works at home, the less reliable or productive they can be in the marketplace. In legal terms, there are missing *property rights* and missing *contracts* between the two institutions. Both of these issues impede the work of the market; they tie down the invisible hand.

3 Health services is an important sector, representing about 16% of the US GDP.

4 To clarify this issue, their experiments (Gneezy et al., 2003) should be augmented to ask the women and the men who participate the amount of time they spend in each of the two institutions. In the case of students, the question may be better posed in terms of the amount of time they expect to spend on each of the two institutions – or the amount of time that their gender role models' – such as parents of teachers – themselves spend at home and in the marketplace.

5 Holmstrom and Milgrom (1991) examine people who share their time among different activities and predict specialisation as does Becker. Their production functions have increasing productivity, and as a result each task is the responsibility of a single person thus predicting hierarchies. Under our conditions, instead, we show that at higher levels of employment equal sharing at home and at the marketplace emerges as the more productive strategy. This increases family welfare, and is more productive in the workplace.

Appendix

Proof of existence of a solution in Proposition 7 (Chichilnisky, 2008b)

We show existence of a solution in a simple case; the most general case requires a fixed point argument. The simplest (non-trivial) case is when $\frac{w_1}{w_2} > M$ as in (12). Under the conditions, as we saw in Proposition 2, women will do all the housework and men will only work in the marketplace. From Chichilnisky [2008b; equation (13)], we obtain the total amount of hours that women work at home, denoted l_2, which as already discussed, produces an externality on the productivity of women at the firm. There is no externality in the case of men, since men do not work at home. Therefore, the total amount of hours that men work at the firm is L_1 and is determined from Chichilnisky [2008b; equation (13)] and so is the marginal productivity $\frac{\partial f}{\partial L_2}$. Since we know the ratio of wages $\frac{w_1}{w_2}$ from Chichilnisky [2008b; equation (13)], we may now derive the number of hours L_2 that women work at the firm together with the value of the externality γ – the two values L_2 and γ_2 must satisfy the following two equations:

$$\frac{w_1}{w_2} = \frac{\frac{\partial f}{\partial L_1}(\gamma).}{\frac{\partial f}{\partial L_2}(\gamma).} \tag{18}$$

and

$$K - L_2 = l_2. \tag{19}$$

To solve the model, we need to find the values of the two variables, γ^* and L_2^*, that satisfy the two equations (18) and (19). One shifts the production function using the externality parameter γ until the two equations are satisfied. At a solution, the productivity of women at the firm will be lower than men's, since women work most of their time at home. The vector $\left(w_1^*, w_2^*, \gamma^*, l_1^*, l_2^*\right)$ is a solution for this economy.