The Gender Gap

Graciela Chichilnisky
Columbia University gc9@columbia.edu
www.chichilnisky.com

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1 Introduction

The gender gap, like the minority achievement gap, has lately become a hot issue. Women are underpaid, undervalued, and overworked across the board. But in our rational economy, what could explain the persistence of this phenomenon? A preferential demand for lower paid women should drive their salaries up until they reach the level of men's. The logic seems impeccable, but it is not borne out by the facts.[1] This article provides an explanation based on the coupling of two distinct institutions: the family and the market. Families are all about *sharing* and using *common property* resources. Firms, instead, use private property to produce private goods, and maximize profits. As far as institutions go, the family and the market could not be further apart, yet they are undeniably intertwined. The way that each responds to the other is critical in understanding and resolving the unequal situation of women in our society.

I hope to explain the seemingly illogical actions of the family-market system by introducing a game between the two components. This game helps to explain the gender gap in salaries, and why men and women allocate time differently between work and home: Inequity at work leads to inequity at home, and vice versa. This vicious circle creates a persistent *gender gap*. It is a rational but undesirable situation, similar to the classical prisioner's dilemma. In economic terms, there are *externalities* between the market and the family because the more a person works at home, the less reliable or productive they can be in the marketplace. In legal terms, there are missing *property rights* and missing *contracts* between the two institutions. Both of these issues impede the work of the market; they tie down the invisible hand..

As in the prisioner's dilemma, our game has a superior outcome that could make everyone better off, but this outcome appears too risky. Under certain circumstances the market and the family can reach a win - win solution that involves equity for women at home and at work, and improves economic productivity and profits.

[1]. The problem persists across all occupations and income levels, and is typically worse at the top. See ([5]), ([12]), ([21]), ([20]), ([18]) ([19]) ([7]), ([14]).

The current, less profitable situation has evolved over time. As women have historically had lower salaries, the family has used more women's labor at home because men can make a higher income in the marketplace. This is a rational response that maximizes family income. At the same time, however, the burden of excessive housework has the effect of lowering women's productivity in the marketplace, by decreasing the time and energy they have available, and their reliability. Women appear to be more 'risky' since they may not be available in case of an emergency. If workers are assets, then women are riskier assets even if they are equally productive. This is used to justify women's lower wages. The problem is more acute in the most demanding and highest paid jobs - thus explaining the Glass Ceiling ([18]) and ([19]).

Recent experiments by Gneezy, Niederle and Rustichini ([13]) show that women perform worse than men in competitive environments. This seems right. Women spend most of their time in the family when their salaries are lower than men's, as shown here. Therefore they can be expected to be better adapted to the cooperative family 'mores' than to the competitive 'mores' of the marketplace. Success within a family requires skill in sharing and using common property to produce public goods, while success in the marketplace requires competitive skills.[2] The experimental evidence appears to confirm the observations made in this article.

In reality, the family's decisions create *externalities* on the firm. From this vicious circle emerges the unequal treatement of women at home and at work. They are trapped in a rational but undesirable situation: *the gender gap*. As previously stated, inequity at home leads to inequity at work and vice versa. Each institution reacts rationally to the other. The government may regulate the workplace, but it cannot regulate the family. But one inequity cannot be solved without the resolution of the other.This is why the gender gap is so difficult to overcome.

Inequity however is not the only solution to the game. I show that there is another rational solution in which women and men are paid the same and share work equally in both institutions. This fair outcome becomes more likely at higher levels of output, when the economy is richer and more productive. Once production exceeds a minimum level a new equilibrium emerges that leads to more welfare at home, more family services, and simultaneously to higher productivity and profits in the marketplace. Fairness is Pareto efficient..

The article is organized as follows. We formalize a toy model of a Walrasian market where women and men have logistic production functions that are typical of 'learning by doing': the more they work the more productive they become, up to a point. This is similar to Becker's [3] well known 1985 article. In his case there is no limit to productivity increases, while here we assume that beyond a certain number of hours per day - say 15 hours - productivity starts to decrease.

[2] To clarify this issue their experiments (??) should be augmented to ask the women and the men who participate the amount of time they spend in each of the two institutions. In the case of students, the question may be better posed in terms of the amount of time they expect to spend on each of the two institutions - or the amount of time that their 'gender role models' - such as parents of teachers - themslves spend at home and in the marketplace.

Becker's economies of scale go on forever, and therefore everyone specializes and no mixed outcomes are possible. Holmstrom and Milgrom (??) [3] examine people who share their time among different activities and also predict that specialization is the answer as does Becker, because their production functions show increasing productivity forever. Each task is the responsibility of a single person, and the result is hierarchies. Under our conditions, instead, we show that at higher levels of employment equity emerges as the more productive strategy. It increases family welfare, and is more productive in the workplace.

This article explains how the coupling of two distinct institutions - markets and the family - can lead to a disproportionate allocation of home responsibilites to women, and simultaneously to the lowering of women's wages. There is a cooperative solution that is better for all, involving equity at home and in the workplace. Why, then, has it not been used? The answer is simple: under our current economic and social conditions, this solution seems riskier. The conclusions suggest how to get there as soon as possible.

1.1 The Firm

The economy has several identical competitive firms producing a good x. A representative firm uses two types of workers, men and women. Their labor is denoted L_1 and L_2 respectively with possibly different wages w_1 and w_2. The firm's production technology is described by a function f

$$x = f(L_1) + f(L_2)$$

The firm's goal is to maximize profits □, namely the difference between the firm's revenues and its costs:

$$Max_{L_1 L_2}(\square) \tag{1}$$

$$= Max_{L_1 L_2}\left[p_x(f(L_1) + f(L_2)) - (w_1 L_1 + w_2 L_2)\right]$$

Since firms are competitive they take the price of good x, p_x and wages w_1 and w_2 as parametrically given. Maximizing profits implies the standard condition that wages must equal the marginal product of labor:

$$w_1 = \frac{\partial f}{\partial L_1} \text{ and } \quad w_2 = \frac{\partial f}{\partial L_2} \tag{2}$$

There are two parameters γ_1 and γ_2 which vary with the person's work at home and influence their productivity in the marketplace. The firm takes these parameters as given; they represent an externality:

$$x = f(L_1, \gamma_1) + f(L_2, \gamma_2)$$

so for each given γ_1, γ_2 profit maximization implies

$$w_1 = \frac{\partial f}{\partial L_1}(\gamma_1) \text{ and } \quad w_2 = \frac{\partial f}{\partial L_2}(\gamma_2)$$

[3] The Holmstrom - Milgrom optimization problem is fundamentally non convex as is Becker's see (??).

1.2 The Family

There are several identical families. Neglecting distributional issues, we refer to a representative family whose welfare derives from family services h, and from the consumption of good x. The family goal is to optimize welfare:

$$Max(U(x,h)) \tag{3}$$

Family services are produced according to a technology g

$$h = g(l_1) + g(l_2) \tag{4}$$

where l_1 and l_2 are the two types of labor in the household, men's and women's respectively. Let K be the total amount of hours that a person can feasibly work in a given period of time, at home and in the market. As an example, in a given day, this could be $K = 14$. When all labor is utilized

$$L_1 = K - l_1 \text{ and } L_2 = K - l_2 \tag{5}$$

The family's *income* equals the wages that its men and women earn in the marketplace plus the firms' profits, since families own the firms. The value of what the family buys $p_x x$ must equal its income:

$$p_x x = w_1 L_1 + w_2 L_2 + \Box \tag{6}$$

where as before profits $\Box$ are the firm's revenues minus its costs:

$$\Box = p_x(f(L_1,\gamma) + f(L_2,\gamma)) - (w_1 L_1 + w_2 L_2) \tag{7}$$

We normalize by assuming that the price of x is one, $p_x = 1$, so that the family's 'budget' equation is

$$x = (6) + (7) = f(L_1) + f(L_2) \tag{8}$$

1.3 The family's trade - off

The family faces a trade - off in deciding whether to use labor at home or in the marketplace. The more is labor used at home, the more family services are produced, but the lower is the family's income and therefor the fewer market goods it consumes. The family has to reach an optimal use of labor at home and in the marketplace to optimize its welfare.

When women and men are paid differently, $w_1 \neq w_2$. the family's decision problem by (5), (4) and (8) is to choose l_1, l_2 to

$$Max_{l_1,l_2} U(f(K - l_1,\gamma) + f(K - l_2,\gamma), g(l_1) + g(l_2)) \tag{9}$$

The family considers the productivity parameters γ_1 and γ_2 as given. From (2) this implies

$$\frac{\partial U}{\partial x}(-w_1) + \frac{\partial U}{\partial h}\frac{\partial g}{\partial l_1} = 0 \tag{10}$$

and

$$\frac{\partial U}{\partial x}(-w_2) + \frac{\partial U}{\partial h}\frac{\partial g}{\partial l_2} = 0$$

Therefore wages determine the productivity of each type of labor at home, and the amount of time each works at home

$$\frac{\partial g}{\partial l_1} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} w_1 \text{ or } w_1 = \frac{\partial g}{\partial l_1}\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} \tag{11}$$

$$\frac{\partial g}{\partial l_2} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} w_2 \text{ or } w_2 = \frac{\partial g}{\partial l_2}\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} \tag{12}$$

Equivalently, we obtain the standard result that the marginal rate of substitution between home services and market goods equals their marginal rates of transformation, which in turn equal the ratio of wages:

$$\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} = \frac{\frac{\partial g}{\partial l_1}}{\frac{\partial g}{\partial l_2}} = \frac{\frac{\partial f}{\partial L_1}}{\frac{\partial f}{\partial L_2}} = \frac{w_1}{w_2} \tag{13}$$

1.4 Public Goods and Common Property resources

Acting as a single single unit, the family makes choices about how to allocate women and men's labor, namely l_1 and l_2. This means that labor is treated as a *common property* of the family. Furthermore, since there is a single welfare level for the entire family, this means that family services are shared as a 'public good' within the family. See also ([1]), ([2])

In summary: *the family produces a public good using common property resources*. Family services are better described as a 'local' public good within the family, because they not shared with other families.

1.5 Learning by doing

Becker [3] pointed out that the more time we spend in a given activity the better we become at doing it. This is called *learning by doing*. It means that marginal productivity g increases with time. Under these conditions, each person in the family (man or woman) should specialize - one should specialize in working at home, and the other in the marketplace:. Both are more productive, at home and in the marketplace, thus increasing family welfare. As a direct consequence of Becker's assumption, when women's salaries are lower than men, women should do all the housework. Men should only work in the marketplace.

Since in reality women's salaries are lower than men's, historically and currently, Becker's assumption leads directly to a division of labor where women stay at home and men work in the marketplace. Under Becker's assumptions the current situation is a rational and efficient solution.

There is indeed learning by doing in our society and therefore Becker's assumption is reasonable, but only up to a point. Human beings need rest after a number of working hours, and this implies a decrease in productivity beyond a certain number of hours of work.

Accordingly, we assume here that that the time derivative of the home production function $\dot{g}$, is initially positive but after a maximum is reached, $\dot{g}$ starts to decrease since humans cannot work productively withour rest.

If $g(t)$ is the amount of h produced with t hours worked, then we may assume that increases in productivity follow a modified quadratic form, increasing initially and then decreasing as was just postulated,

$$\dot{g}_t = H(g_t) = \beta g - \gamma g^2 \text{ with } \beta, \gamma > 0$$

This equation integrates to yield the classic logistic curve that is used often to describe the evolution of biological populations over time:

$$g(t) = \frac{\beta g_0}{\gamma z_0 + (\beta - \gamma z_0)\exp(-\beta t)}$$

The logistic function $g(t)$ has an *inflection point:* e.g. when $g_0 = 1$, the inflection point is at $g = \frac{\beta}{\gamma}$. Assuming that $g_0 = 1$, the evolution over time of productivity of labor increases with the number of hours worked, until it reaches a maximum increase at $g = \frac{\beta}{\gamma}$ and declines afterwords. The second derivative is positive until the inflection point, and negative afterwards. The graph of the function is therefore convex until the value $\frac{\beta}{\gamma}$ and it is concave thereafter.

The convex part is similar to Becker's assumption and yields similar results. On the other hand the concave part, which occurs after the inflection point is reached yields very different results as is shown below. The inflection point determines a change from one regime to the other.

Assumption 1. In the following we assume that production has reached the inflection point at home and at the marketplace, an assumption that seems to tally with the evidence. We describe this as having achieved *higher levels of output.*

1.6 Equity at home improves welfare

Proposition 1. At higher levels of output, equity benefits the family.

Distributing home labor equally between men and women produces more household services for the same total labor. Formally, if

$$\frac{l_1 + l_2}{2} > \frac{\beta}{\gamma}$$

then

$$l_1 \neq l_2 \Rightarrow g(l_1) + g(l_2) < 2g(\frac{l_1 + l_2}{2})$$

Proof:

$$2g(\frac{l_1 + l_2}{2}) > g(l_1) + g(l_2) \Leftrightarrow$$

$$g(\frac{l_1 + l_2}{2}) > \frac{g(l_1)}{2} + \frac{g(l_2)}{2},$$

which is the definition of concavity. Above its inflection point the logistic curve g is concave since its second derivative is negative, proving the inequality. Equity is a more efficient use of resources at home whenever

$$\frac{l_1 + l_2}{2} > \frac{\beta}{\gamma},$$

as we wished to prove.■

1.7 Inequity at work leads to inequity at home

There is historic difference in the average pay of men and women, about 25% or 30% in the US. What is the optimal response by the family to this inequity, in terms of allocating labor at home? The following proposition provides a response:

Proposition 2. Inequity at work leads to inequity at home
When women are paid less than men in the marketpace, $w_1 > w_2$, the family's optimal response is that women should work longer hours at home than men. When the difference in wages is large enough, $\frac{w_1}{w_2} > M = \frac{\sup \frac{\partial g}{\partial l_1}}{\inf \frac{\partial g}{\partial l_2}}$ it is optimal for the family that women should do all the housework, and men should work only in the marketplace.

Proof: From (3) and (9) the family's goal is

$$Max_{l_1,l_2} U(f(K - l_1) + f(K - l_2), g(l_1) + g(l_2))$$

From (13)

$$\frac{\frac{\partial g}{\partial l_1}}{\frac{\partial g}{\partial l_2}} = \frac{w_1}{w_2}$$

so that at an optimum

$$w_1 > w_2 \text{ implies } \frac{\partial g}{\partial l_1} > \frac{\partial g}{\partial l_2}.$$

Therefore women work up to the point where their marginal productivity is lower than men's. As we saw in Section 1.5, when $g(t) > \frac{\beta}{\gamma}$,. the marginal productivity of labor $\frac{\partial g}{\partial l_2}$ is a decreasing function of the time allocated, so that lower productivity means longer hours for women at home.

When the ratio of salaries exceeds M the ratio of the supremum and the infimum productivity of g, namely when

$$\frac{w_1}{w_2} > M = \frac{\sup \frac{\partial g}{\partial l_1}}{\inf \frac{\partial g}{\partial l_2}} \tag{14}$$

it is optimal that women should completely specialize in housework. This completes the proof.■

Proposition 2 implies that it is always optimal for the family to use more women labor at home when they have lower salaries than men. If women housework's hours are less than the maximum feasible, K, then it would be rational that women should also work in the marketplace in addition to their work at home - at their reduced salaries. Furthermore, when salary differentials are large enough, it is optimal for the family that women do all the housework and that they work also in the marketplace at reduced salaries while men, on the other hand, work only in the marketplace and at higher salaries.

The logic of the situation and (13) imply that when $w_1 > w_2$, then women's marginal productivity is lower than men's at home and also in the marketplace. When production functions f and g are concave, this implies in turn that women work more hours than men at home and also in the marketplace, because marginal productivity decreases with the time worked, so that:

$$L_1 > L_2 \text{ and } l_1 > l_2 \tag{15}$$

However

$$L_1 = K - l_1 \text{ and } L_2 = K - l_2$$

implies

$$L_1 > L_2 \Rightarrow l_2 > l_1 \tag{16}$$

How to reconcile (15) and (16)? In the next section we show that the externality that the home produces on the firm, namely the parameters γ_1 and γ_2 reconcile these two apparently divergent inequalities..

1.8 Externalities: inequity at home reduces women's productivity in the market

As already pointed out, the amount of work that a person performs at home has an impact on their productivity in the marketplace. The first hour that a woman works at the firm may be the 6th hour of work that day, since she may have worked already 5 hours at home.

Yet the number of hours that a person works at home are not known to the firm, nor can the firm control them. This is an *externality* that the family causes the firm. Formally, l_1 and l_2 are treated as parameters by the firm even though they have an impact on the firm through worker's productivity.. These observations may be formalized as follows:

Assumption 1: There exists a parameter $\gamma > 0$ representing an 'externality' on the firm so that for $i = 1,2$

$$\text{For } i = 1,2 \qquad \frac{\partial f}{\partial L_i} = \frac{\partial f}{\partial L_i}(\gamma) \text{ where } \frac{\partial^2 f}{\partial \gamma \partial L_i} < 0.$$

A simple example of this phenomenon would be

$$f(L_i) = \gamma(l_i) L_i^\alpha$$

where

$$\gamma = \gamma(l_i) \text{ and } \partial\gamma/\partial l_i < 0.$$

Under Assumption 1 above:

Proposition 3: Inequity at home leads to lower productivity of women at work, and to lower salaries for women

This is an immediate consequence Assumption 1 and (13).∎.

The productivity of women in the marketplace depends on the amount of time they work at home. This breaks the symmetry between productivity at work and hours worked. Even if the production function f is concave, those who spend more time working at home could have lower productivity in the marketplace while working fewer hours than the rest. The production function f depends not only on L but also on l and at higher levels of l, the graph of $f(L)$ shifts downwards due to the externality. This resolves the apparent conflict in (15) and (16) above.

1.9 Inequity lowers family welfare

We saw that inequity at work leads to inequity at home and that inequity at home reduces productivity at work for those working longer hours at home. If women are subject to this inequity, then obviously they are worse off under these conditions. Is it possible however that the family as a whole is better off? The following proposition provides a response.

Proposition 4: At higher levels of output, inequity lowers family welfare, decreasing both family services h and the family's consumption of market goods x.

Proof: We have alreadyb shown that, under the conditions, the family produces more home services h with the same total amount of labor if the work load is distributed equally between the two genders. Namely when $\frac{l_1+l_2}{2} > \frac{\beta}{\gamma}$

$$l_1 \neq l_2. \Rightarrow 2g(\frac{l_1 + l_2}{2}) > g(l_1) + g(l_2)$$

Inequity leads to less family services h.

Yet it is still possible that inequity at home could increase family income sufficiently to compensate for the loss in family services. We show that this is not possible under the conditions.

By definition, inequity at home means $l_1 < l_2$

$$\text{which implies } L_1 = K - l_1 > L_2 = K - l_2$$

This under the conditions implies that women's marginal productivity at work is lower than men's, see (13) above. Since the firm has a logistic production function f then for the same total amount of labor $L_1 + L_2$ an equal workload among women and men increases total output:

$$2f(\frac{L_1 + L_2}{2}) > f(L_1) + f(L_2) \quad \text{when } L_1 \neq L_2$$

as shown in Proposition 1 above. Therefore the total production of market goods x is lower than when men and women share work equally. Since all production is consumed by families, the family consumes less market goods x as well as fewer family services. Therefore inequity at home lowers the family's welfare as we wished to prove.■

1.10 Inequity leads to lower output and lower profits

Proposition 5. At higher output levels, inequity reduces the firm's output and lowers its profits

Proof: We saw in Proposition 4 that under the conditions, inequities decrease the market's output of x. For the same total amount of work the production of the firms is higher when men and women divide equally the work load:

$$2f(\frac{L_1 + L_2}{2}) > f(L_1) + f(L_2) \text{ when } L_1 \neq L_2,$$

This proves the first part of the proposition. It remains to consider the impact of inequity on profits, namely on the function

$$\Pi(L_1, L_2) = f(L_1) + f(L_2) - w_1 L_1 - w_2 L_2$$

We wish to compare

$$\Pi(L_1) + \Pi(L_2) \quad \text{with} \quad 2\Pi(\frac{L_1 + L_2}{2})$$

By concavity (since we are above the inflection point of f) profits increase with the level of output, namely

$$\frac{\partial \Pi}{\partial x} > 0$$

Since equity increases output, and profit is an increasing function of output, it follows that equity increases profits as well. Equivalently, inequity decreases output and profits as we wished to prove..■

1.11 A Nash - Walrasian solution

This section describes the functioning of the economy as a whole. The economy consists of a Walrasian market where firms maximizes profits, and of families that produce public goods using common property resources, maximizing welfare. There are three traded goods in the economy: the market good x, woment labor, and men's labor. We normalized the price of x so that $p_x = 1$.

Recall that the family is not Walrasian; its services h are shared among the members, which makes them similar to (local) public goods. Furthermore, the resources such as labor l_1 and l_2 that are used to produce h are allocated by common decision within the family so as to maximize the family's welfare. Therefore the family treats resources as common property. Additionally the family produces an externality on the firm γ which depends on the hours that men and women work at home, $\gamma = \gamma(l_i), i = 1, 2$. There are no benchmark models to analyze the functioning of such a mixed economy.

We need some definitions.

Definition: If $w_1 \neq w_2$ we say that *the market is unfair.* If $w_1 = w_2$ we say that *the market is fair.*

Definition: If $l_1 \neq l_2$ we say that *the family is unfair* and if $l_1 = l_2$ we say that *the family is fair.*

Proposition 6: Given wages for the two types of labor w_1 and w_2 from the family's welfare optimization behavior (3)it is possible to determine the amount of family services it produces, the employment of men's and women's labor at home, $l_1$and l_2, the offer of labor of the two types to the marketplace, $K - l_1$ and $K - l_2$, the family's demand for market goods, the family's income, its welfare level, and the value of the externality parameters $\gamma_1(l_1)$ and $\gamma_2(l_2)$ which modify the firm's production function.. On the other hand, the firm has expected values for the parameters $\gamma_1{}^e$and γ_2^e and from the firm's profit maximization behavior (1) it is possible to determine the amount of labor the firm wishes to employ (men and women), how much it produces, what are its profits, and the productivity of its labor.

This is a standard microeconomic exercise.■

In Proposition 6 the family and the firm may have contradictory goals in terms of the productivity parameters $\gamma_1{}^e$and γ_2^e, the market goods produced and consumed and people employed. A *solution* for this economy arises when firms and families behave consistently:

Definition 1. A *solution* for this economy consists of wages for men and for women w_1^*, w_2^* and expected values of the parameters $\gamma_1^e\ \gamma_2^e$ leading to consistent behavior by the family and the firm. The levels of employment and consumption that derive from profit optimization by the firm and from welfare optimization by the family, clear all three markets, and the value of the externality produced by the family on the firm equal the values expected by the firm.

In particular:

(0) Expectations are confirmed

$$\gamma(l_1) = \gamma_1^e \quad \text{and } \gamma(l_2) = \gamma_2^e$$

(1) Supply of men's labor equals demand for men's labor by the firm

$$L_1^D(w_1, w_2) = N.\arg\max \square(w_1, w_2) = L_1^S(w_1, w_2) = 15 - l_1(w_1, w_2) \qquad (17)$$

(2) Supply of female labor equals demand of women's labor by the firm

$$L_2^D(w_1, w_2) = N.\arg\max \square(w_1, w_2) = L_2^S(w_1, w_2) = 15 - l_2(w_1, w_2) \qquad (18)$$

and

(3) Supply by the firm of x equals the family's demand for x,

$$x^S(w_1, w_2) = f(L_1^D(w_1, w_2), L_2^D(w_1, w_2)) = x^D(w_1, w_2) = w_1 L_1 + w_2 L_2 + \square \qquad (19)$$

The existence of a solution shows that the model as postulated is internally consistent.

Proposition 7: There exists a solution for this economy.
Proof: In the Appendix.

1.12 The Market - Family Game

This section defines a game with two players, the market and the family. The *market's objective* is to maximize profits as defined in (1). The *family's objective* is to maximize welfare as defined in (3). The players choose their strategies to achieve their goals. The *market's strategy* is to set wages for men and for women, w_1 and w_2, and expectations about their productivity γ_1^e and γ_2^e while the *family's strategy* is to allocate labor at home among men and women, l_1 and l_2.

Definition. A **Nash equilibrium** is a set of strategies for the market and for the family $(w_1^*, w_2^*, \gamma_1^e, \gamma_2^e, l_1^*, l_2^*)$ leading to a solution for the economy in which each player reacts optimally to the other's strategy, so neither has an incentive to deviate.

Proposition 8. At high levels of output:

1. A Nash equilibrium where women have lower salaries. The family reacts by allocating more house work to women. Conversely, at a Nash equilibrium where the family allocates more housework to women, women productivity is lower in the marketplace and they receive lower salaries than men. This Nash equilibrium is called *unfair-unfair*.

2. A Nash equilibrium where women have the same salaries as men. Women have the same productivity. The family reacts by sharing equally hosework between men and women. Conversely, at a Nash equilibrium where women and men share housework equally, their wages in the marketplace are the same as men's. This is a *fair-fair* Nash equilibrium.

3. The *unfair-unfair* Nash equilibrium is Pareto inferior. The *fair - fair* Nash equilibrium is Pareto efficient, but it is riskier..

Proof: When women have the same salaries as men, both bring the family the same income for the same hours in the markeplace. By (13) their productivity is the same at an optimum, and given the assumptions, it is more productive for both men and women to work the same hours in the markeplace. At the same time, by Proposition 1 women work at home the same number of hours as men, since under the conditions, sharing work equally at home provides more family services for the same total amount of labor.

Reciprocally, when women and men share work equally at home, then it is optimal for the firm to pay both equally from (13). The *fair - fair* pair of strategies just described is a Nash equilibrium of the market- family game because when following such a pair of strategies, each player is responding optimally to the others' move.

At a Nash equilibrium where women's salaries are inferior to men's, it is optimal for the family to choose an unfair distribution of household work by Proposition 2. Women work more at home, and their productivity at home is lower as shown in Proposition 2 and in Section 1.9, and so is their productivity at work by (13). This is an *unfair-unfair* Nash equilibrium, with both players responding optimally to each other. Nevertheless, it is a Pareto inferior solution.

The *first fair-fair* equilibrium is Pareto optimal. The following section illustrates why the fair-fair equilibrium is riskier under the conditions. This completes the proof. ■

1.13 A matrix game

The matrix below illustrates a game where the horizonal strategies represent the market's and the vertical represent the family's. The payoffs for the market and sub - indexed 1 and those for the family are sub- indexed 2.

$$\begin{array}{cc} & \begin{array}{cc} w_1 \neq w_2 & w_1 = w_2 \end{array} \\ \begin{array}{c} l_1 \neq l_2 \\ l_1 = l_2 \end{array} & \begin{pmatrix} (A_1, A_2) & (C_1, D_2) \\ (D_1, C_2) & (B_1, B_2) \end{pmatrix} \end{array}$$

In this matrix game, Proposition 8 can be summarized by the inequalities

$$C_1 < A_1 < B_1$$

and

$$C_2 < A_2 < B_2$$

when (A_1, A_2) is the outcome of the unfair-unfair Nash equilibrium, (B_1, B_2) is the outcome of the fair fair Nash equilibrium. The *fair-fair* Nash equilibrium is Pareto efficient because $A_1 < B_1$ and $A_2 < B_2$.

The Pareto efficient Nash equilibrium is more risky, because $C_1 < A_1$ so if the market plays *fair* but the family plays *unfair* the market will be worse off, this is Proposition 3. Conversely, $C_2 < A_2$ implies that the family will be worse off if it playes *fair* while the market plays *unfair*, by Proposition 2.

1.14 The family - market game is similar to the Prisioner's dilemma.

The matrix presented above is similar to that of the 'prisioner's dilemma game' when in addition to the inequalities:

$$C_1 < A_1 < B_1 < D_1$$

and

$$C_2 < A_2 < B_2 < D_2$$

the two players are symmetrically situated, so that

$$A_1 = A_2, B_1 = B_2, C_1 = C_2, D_1 = D_2$$

A numerical example of the prisioner's dilemma is

$$\begin{pmatrix} 5,5 & 3,10 \\ 10,3 & 9,9 \end{pmatrix}$$

while a numerical example of our situation need not be symmetrical - for example

$$\begin{pmatrix} 5,6 & 3,10 \\ 9,4 & 8,9 \end{pmatrix}$$

where

$$\begin{aligned} A_1 &= 5 \quad & A_2 = 6 \\ B_1 &= 8 \quad & B_2 = 9 \\ C_1 &= 3 \quad & C_2 = 4 \\ D_1 &= 9 \quad & D_2 = 10 \end{aligned}$$

1.15 Conclusions

The coupling of two distinct institutions - the market and the family - can lead to a disproportionate allocation of home responsibilities to women, and simultaneously to the lowering of women's wages. We showed that there is a cooperative solution that is better for all, involving equity at home and in the workplace, but it seems riskier. The risks derive from missing contracts between the family and the marketplace. The family loses if it plays fair when the market doesn't, and vice versa.([9]) ([10]) ([11]).

What social institutions can help resolve this problem? Waldfogel and others ([22]) have considered similar issues.

A prenuptial agreement that specifies women's and men's roles in the family could be a start. It should have penalty attached if the parties default from what was promised. Using such a legal agreement , women can present themselves at work as fully able to deliver so a fair employer is not misled about the nature of the labor it hires.

Similarly, strengthening equal pay provisions in the marketplace should support the execution of these prenuptial agreements. This requires enforcing the Equal Pay Act - and perhaps making this enforcement contingent on the availability of the prenuptial agreement just discussed. This way the firms would not risk being penalized for playing fair.

Other solutions to the prisioner's dilemma have been proposed over the years, most of them encourage cooperation among the players. Often this requires repeated games among the players, which is not realistic in the case of marriage ([16]). In any case, any solution that encourages a cooperative outcome between the family and the market will benefit both. The moral of this article is that equity may appear to be riskier - and indeed, it may be- but it is after all the Pareto efficient allocation. Room should be made for the missing contracts between the players - the market and the family - that take advantage of the existence of a win-win solution, making everyone better off.

1.16 Appendix

Proof of existence of a solution in Proposition 7.

We show existence of a solution in a simple case; the most general case requires the use of a fixed point argument.. The simplest (non trivial) case is when $\frac{w_1}{w_2} > M$ as defined in (14). Under the conditions, as we saw in proposition 2, women will do all the housework and men will only work in the marketplace. From (13) we obtain the total amount of hours that women work at home, denoted l_2, which as already discussed, produces an externality on the productivity of women at the firm. There is no externality in the case of men, since men do not work at home. Therefore the total amount of hours that men work at the firm is L_1 and is determined from (13) and so is the marginal productivity $\frac{\partial f}{\partial L_2}$. Since we know the ratio of wages $\frac{w_1}{w_2}$ from (13) we may now derive the number of hours L_2 that women work at the firm together with the value of the externality $\gamma-$ the two values L_2 and γ_2 must satisfy the following two equations

$$\frac{w_1}{w_2} = \frac{\frac{\partial f}{\partial L_1}(\gamma).}{\frac{\partial f}{\partial L_2}(\gamma).} \tag{20}$$

and

$$K - L_2 = l_2. \tag{21}$$

To solve the model we need to find the values of the two variables, γ^* and L_2^*, that satisfy the two equations (20) and (21). One shifts the production function using the externality parameter γ until the two equations are satisfied. At a solution, the productivity of women at the firm will be lower than men's, since women work most of their time at home. The vector $(w_1^*, w_2^*, \gamma^*, l_1^*, l_2^*)$ is a solution for this economy. ■

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