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The Gender Gap

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Abstract

The author explains the "gender gap" as a Nash equilibrium of a game with incomplete information about women's work at home and in the marketplace. Expectations about women's lower wages leads to the overutilization of women in the household, and this, in turn, leads to lower productivity and lower wages for women in the marketplace. The situation is rational but generally Pareto inferior. With logistic learning by doing, at high levels of skill there is a Pareto-superior equilibrium, where men and women share efforts equally at home and receive the same pay in the marketplace, firms enhance their profits, and there is more welfare at home. Inequity at home breeds inequity in the marketplace and, reciprocally, inequity in the marketplace leads to inequity at home, causing a persistent gender gap. Appropriate contracts may be needed to implement the superior solution, since generally governments do not intervene in family matters.

1. Introduction

The gender gap, like the minority achievement gap, has lately become a hot issue. Women are underpaid, undervalued, and overworked across the board. But in our rational economy, what could explain the persistence of this phenomenon? In uniform populations, preferential demand for lower paid women should drive their salaries up until they reach the level of men's. The logic seems impeccable, but it is not borne out by the facts.[1] This article provides an explanation based on the coupling of two institutions: the family and the market. Families are about *sharing* and using *common property* resources. Firms, instead, use private property to produce private goods and maximize profits. As far as institutions go, the family and the market could not be further apart, yet they are undeniably intertwined. The way that each responds to the other is critical in understanding and resolving the unequal situation of women in our society.

I explain the seemingly illogical actions of the family-market system by introducing a game with incomplete information, involving two components—the family and the firm. The firm does not know the hours worked by women in the household, and household work creates a negative externality on women's productivity in the firm. The game helps to explain the gender gap in salaries, and why men and women allocate time differently between work and home: inequity at work leads to inequity at home, and vice versa. This vicious circle creates a persistent *gender gap*. The government may regulate the workplace, but it does not normally regulate the family. Therefore, since one inequity cannot be solved without the other, the gender gap becomes very difficult to overcome.

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The current situation has evolved over time. Women had lower salaries historically. They performed most housework because men could make a higher income working in the marketplace. Under these conditions, this division of labor is a rational way to maximize family income. However, the burden of excessive housework decreases the time and the energy that women can bring to the marketplace. The family produces *externalities* on the firm.[2] In such a situation women appear to be more *risky* because they may not be available in case of emergencies; for example, if they act as the main providers of the medical needs of the family.[3] If workers are assets, then women are *riskier assets* even when they are equally productive. This riskiness is in turn used to justify women's lower wages, closing the vicious circle. From this game between the family and the marketplace, the gender gap emerges as a rational and stable Nash equilibrium. It is a rational but undesirable situation for the players, one that is similar to the classical prisoner's dilemma.

Women's riskiness is felt most acutely in the most demanding and highest paid jobs where 24/7 availability is often required—thus creating a larger gap in women participation and salaries at the highest levels—explaining the glass ceiling (see Meyersson Milgrom et al., 2001 and Meyersson Milgrom and Petersen, 2003).

The empirical and experimental evidence appears to confirm these observations. Recent empirical research in 28 nations confirms that women work at home at least 70% more than men (Davis, Greenstein and Marks, 2007),[4] and the specific results in this article have been tested empirically with US household data (Chichilnisky and Schachmurove, 2006). Additionally, Bonke et al. (2005) found that larger differences between men and women's work at home are associated with larger differences in market salaries, and recent experiments by Gneezy et al. (2003) show that women perform worse than men in competitive environments. Both make sense. Due to their lower salaries, women spend more time working at home where the most important skills involve sharing and cooperation. Success in the marketplace requires instead more competitive skills.[5] One can therefore expect women to adapt to the cooperative family "mores," while men adapt instead to the competitive "mores" of the marketplace.

This article formalizes a toy game where women and men share their time between the family and a Walrasian market economy. They *learn by doing*, in the sense that the more they work the more productive they are. This follows Becker's classic 1985 article, which provides the standard argument for specialization of women and men, at home or in the marketplace, respectively.[6] In contrast to Becker's assumption, however, I follow Arrow's 1962 seminal article where he introduced *learning by doing*. In Arrow's formulation there are *decreasing returns* after a certain number of hours per day. Somewhat surprisingly, I show that Arrow's assumption reverses Becker's findings in the sense that specialization is no longer necessary for efficiency. There is now another, rational, solution in which women and men are paid the same and share the work equally in both institutions. This fair outcome emerges at higher levels of output, when the economy is richer and more productive. Once production exceeds a minimum level we enter Arrow's regime where the learning curve is concave rather than convex as it is initially, and as it is in Becker's work. I show that a new equilibrium emerges that leads to more welfare at home, to more family services, and simultaneously to higher productivity and profits in the marketplace. Inequity is no longer the only solution—now fairness is Pareto efficient.

If such equitable solutions exist, one may ask, why aren't they used more often? The answer is simple: under our current economic and social conditions, the equitable solution seems riskier, as is the optimal solution in the prisoner's dilemma. There are missing contracts between the players. Equal treatment in the family depends on equal treatment in the marketplace and vice versa[7]—but neither institution can safely depend on the other. In the conclusions I suggest the introduction of new contractual arrangements between the family and the marketplace that can overcome this lack of contracts and help reach efficient and equitable social solutions, as soon as possible.

2. The Firm

The economy has several identical competitive firms producing a good x. A representative firm uses two types of workers, men and women. Their labor is denoted L_1 and L_2 respectively with possibly different wages w_1 and w_2. The firm's production technology is described by a function f:

$$x = f(L_1) + f(L_2),$$

that is further specified below.

The firm's goal is to maximize profits π, namely the difference between the firm's revenues and its costs:

$$\text{Max}_{L_1L_2}(\pi) = \text{Max}_{L_1L_2}[p_x(f(L_1) + f(L_2)) - (w_1L_1 + w_2L_2)]. \tag{1}$$

Since firms are competitive they take the price of good x, p_x and wages w_1 and w_2 as parametrically given. Maximizing profits implies the standard condition that wages must equal the marginal product of labor:

$$w_1 = \frac{\partial f}{\partial L_1} \quad \text{and} \quad w_2 = \frac{\partial f}{\partial L_2}. \tag{2}$$

There are two parameters γ_1 and γ_2 which vary with the person's work at home and influence his or her productivity in the marketplace. The firm takes these parameters as given and has no way to find out about them; they represent an "externality" to the firm:

$$x = f(L_1, \gamma_1) + f(L_2, \gamma_2).$$

Since the firm cannot observe nor gather information (in the US, by law) about the extent of women's household work, for each given γ_1, γ_2 profit maximization by the firm implies

$$w_1 = \frac{\partial f}{\partial L_1}(\gamma_1) \quad \text{and} \quad w_2 = \frac{\partial f}{\partial L_2}(\gamma_2).$$

3. The Family

There are several identical families. Neglecting distributional issues, we refer to a representative family whose welfare derives from family services h, and from the consumption of good x. The family goal is to optimize welfare:

$$Max(U(x, h)). \tag{3}$$

Family services are produced according to a technology g:

$$h = g(l_1) + g(l_2), \tag{4}$$

where l_1 and l_2 are the two types of labor in the household, men's and women's, respectively. Let K be the total amount of hours that a person can feasibly work in a given period of time, at home and in the market. As an example, in a given day, this could be $K = 15$. When all labor is utilized

$$L_1 = K - l_1 \quad \text{and} \quad L_2 = K - l_2. \tag{5}$$

The family's *income* equals the wages that its men and women earn in the marketplace, plus the firm's profits, since families own the firms. The value of what the family buys in the marketplace, $p_x x$, must equal its income. The budget equation is:

$$p_x x = w_1 L_1 + w_2 L_2 + \pi, \tag{6}$$

where, as before, profits π are the firm's revenues minus its costs:

$$\pi = p_x(f(L_1, \gamma_1) + f(L_2, \gamma_2)) - (w_1 L_1 + w_2 L_2). \tag{7}$$

We normalize by assuming that the price of x is one, $p_x = 1$, so that the family's "budget" equation from (6) and (7) is:

$$x = f(L_1) + f(L_2). \tag{8}$$

4. The Family's Trade-off

The family faces a trade-off in deciding whether to use labor at home or in the marketplace. The more labor is used at home, the more family services are produced, but the lower is the family's income and therefore the fewer market goods it consumes. The family has to reach an optimal use of labor at home and in the marketplace to optimize its welfare.

When women and men are paid differently, $w_1 \neq w_2$, the family's decision problem, by (5), (4), and (8), is to choose l_1, l_2 to

$$\text{Max}_{l_1,l_2} U(f(K - l_1, \gamma_1) + f(K - l_2, \gamma_2), g(l_1) + g(l_2)) = \text{Max}\, U(x, h). \tag{9}$$

The family considers the productivity parameters γ_1 and γ_2 as given. From (2) this implies

$$\frac{\partial U}{\partial x}(-w_1) + \frac{\partial U}{\partial h}\frac{\partial g}{\partial l_1} = 0, \tag{10}$$

and

$$\frac{\partial U}{\partial x}(-w_2) + \frac{\partial U}{\partial h}\frac{\partial g}{\partial l_2} = 0.$$

Therefore wages determine the productivity of each type of labor at home, and the amount of time each works at home:

$$\frac{\partial g}{\partial l_1} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} w_1 \quad \text{or} \quad w_1 = \frac{\partial g}{\partial l_1}\frac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial x}}, \tag{11}$$

$$\frac{\partial g}{\partial l_2} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} w_2 \quad \text{or} \quad w_2 = \frac{\partial g}{\partial l_2}\frac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial x}}, \tag{12}$$

Equivalently, we obtain the standard result that the marginal rates of transformation, equals the ratio of wages:

$$\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} = \frac{\frac{\partial g}{\partial l_1}}{\frac{\partial g}{\partial l_2}} = \frac{\frac{\partial f}{\partial L_1}}{\frac{\partial f}{\partial L_2}} = \frac{w_1}{w_2}. \tag{13}$$

5. Public Goods and Common Property Resources

Acting as a single unit, the family makes choices about how to allocate women and men's labor, namely l_1 and l_2. This means that household labor is treated as a *common property* of the family. Furthermore, since there is a single welfare level for the entire family, this means that family services are shared as a "public good" within the family. See also Apps and Rees (1997) and Aronsson et al. (2001).

In summary: *the family produces a public good using common property resources*. Family services are better described as a "local" public good within the family, because they are not shared with other families.

6. Learning by Doing: Empirical Evidence of Logistic Curves

Arrow (1962) pointed out that the more time we spend in a given activity the better we become at doing it. This is called *learning by doing*. Becker (1985) assumed that marginal productivity g increases with time. Under these conditions, each person in the family (man or woman) should specialize—one should specialize in working at home, and the other in the marketplace. Both are more productive, at home and in the marketplace, thus increasing family welfare. As a direct consequence of Becker's assumption, when women's salaries are lower than men's, women should do all the housework. Men should only work in the marketplace.

Since in reality women's salaries are lower than men's, historically and currently, Becker's assumption leads directly to a division of labor where women stay at home and men work in the marketplace. Under Becker's assumption the current situation is a rational and efficient solution.

There is indeed learning by doing in our society, and therefore Becker's assumption is reasonable, but only up to a point. Human beings need rest after a number of working hours, and this implies a decrease in marginal productivity beyond a certain number of hours of work. Accordingly, we assume here that the time derivative of the home production function $\dot{g}$ is initially positive, but, after a maximum is reached, $\dot{g}$ starts to decrease since humans cannot work productively without rest.

If $g(t)$ is the amount of h produced with t hours worked, then we may assume that increases in productivity follow a modified quadratic form, increasing initially and then decreasing as was just postulated:

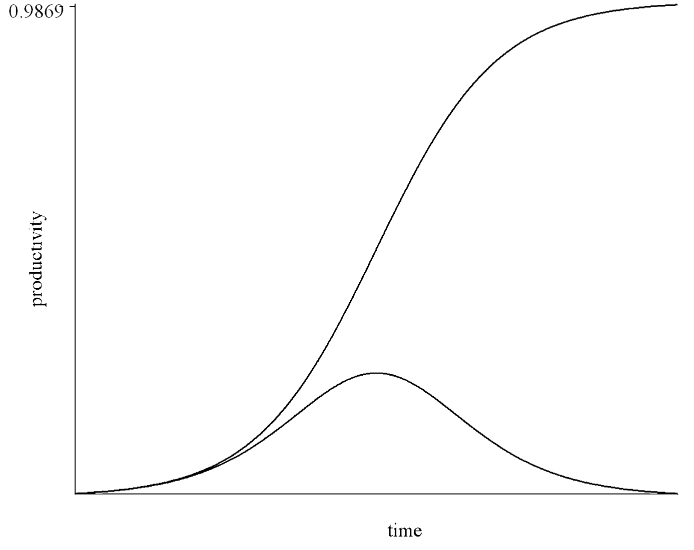
$$\dot{g}_t = H(gt) = \beta g - \gamma g^2 \quad \text{with} \quad \beta, \gamma > 0. \tag{14}$$

This equation integrates to yield the classic logistic curve that is used often, for example to describe the evolution of biological populations over time:

$$g(t)=\frac{\beta g_0}{\gamma z_0+(\beta-\gamma z_0)\exp(-\beta t)}.$$

The logistic function $g(t)$ has an *inflection point*: e.g. when $g_0 = 1$, the inflection point is at $g=\frac{\beta}{\gamma}$. Assuming that $g_0 = 1$, the evolution over time of marginal productivity of labor increases with the number of hours worked, until it reaches a maximum increase at $g=\frac{\beta}{\gamma}$ and declines afterwards. The second derivative is positive until the inflection point, and negative afterwards. The graph of the function is therefore convex until the value $\frac{\beta}{\gamma}$ and is concave thereafter.

The convex part is similar to Becker's assumption and yields similar results. On the other, hand the concave part, which occurs after the inflection point is reached, yields very different results, as is shown below. The inflection point determines a change from one regime to the other; it appears in the diagram below as the maximum of the quadratic curve, which is the derivative of g:



Assumption 1. *In the following we assume that production has reached the inflection point at home and at the marketplace, an assumption that seems to tally with the evidence. We describe this situation as having achieved* higher levels of output.

7. Equity at Home Improves Welfare

Proposition 1. *At higher levels of output, equity benefits the family.*

Distributing home labor equally between men and women produces more household services for the same total labor. Formally, if

$$\frac{l_1+l_2}{2}>\frac{\beta}{\gamma},$$

where β and γ are as in (14) above then

$$l_1 \neq l_2 \Rightarrow g(l_1) + g(l_2) < 2g\left(\frac{l_1 + l_2}{2}\right).$$

PROOF.

$$2g\left(\frac{l_1 + l_2}{2}\right) > g(l_1) + g(l_2) \Leftrightarrow g\left(\frac{l_1 + l_2}{2}\right) > \frac{g(l_1)}{2} + \frac{g(l_2)}{2},$$

which follows from the definition of concavity. Above its inflection point the logistic curve g is concave since its second derivative is negative, proving the inequality. Formally, equity is a more efficient use of resources at home whenever

$$\frac{l_1 + l_2}{2} > \frac{\beta}{\gamma}.$$

□

8. Inequity at Work Leads to Inequity at Home

There is a historic difference in the average pay of men and women, about 25% or 30% in the US. What is the optimal response by the family to this inequity, in terms of allocating labor at home? The following proposition provides a response:

PROPOSITION 2. *Inequity at work leads to inequity at home.*

When women are paid less than men in the marketplace, $w_1 > w_2$, the family's optimal response is that women should work longer hours at home than men. When the difference in wages is large enough, $\frac{w_1}{w_2} > M = \frac{\sup \frac{\partial g}{\partial l_1}}{\inf \frac{\partial g}{\partial l_2}}$, it is optimal for the family that women should do all the housework, and men should work only in the marketplace.

PROOF. From (3) and (9) the family's goal is

$$\text{Max}_{l_1, l_2} U(f(K - l_1) + f(K - l_2), g(l_1) + g(l_2)).$$

From (13)

$$\frac{\frac{\partial g}{\partial l_1}}{\frac{\partial g}{\partial l_2}} = \frac{w_1}{w_2},$$

so that at an optimum

$$w_1 > w_2 \quad \text{implies} \quad \frac{\partial g}{\partial l_1} > \frac{\partial g}{\partial l_2}.$$

Therefore women at home work up to the point where their marginal productivity delta g over delta l subscript 2, as in line below is lower than men's. As we saw in section 6, when $g(t) > \frac{\beta}{\gamma}$, the marginal productivity of labor $\frac{\partial g}{\partial l_2}$ is a decreasing function of the time allocated, so that lower productivity means longer hours for women at home.

Finally, when the ratio of salaries exceeds M, the ratio of the supremum and the infimum productivity of g, namely when

$$\frac{w_1}{w_2} > M = \frac{\sup \frac{\partial g}{\partial l_1}}{\inf \frac{\partial g}{\partial l_2}}, \tag{15}$$

it is optimal that women should completely specialize in housework. □

Proposition 2 implies that it is always optimal for the family to use more women's labor at home when they have lower salaries than men. If women's housework hours are less than the maximum feasible, K, then it would be rational that women should also work in the marketplace in addition to their work at home—at lower salaries than men. Furthermore, when salary differentials are large enough, it is optimal for the family that women do all the housework and that they work also in the marketplace at reduced salaries, while men, on the other hand, work only in the marketplace and at higher salaries.

The logic of the situation, and (13), imply that when $w_1 > w_2$, then women's marginal productivity is lower than men's at home and also in the marketplace. When production functions f and g are concave, this implies in turn that women work more hours than men at home and also in the marketplace, because marginal productivity decreases with the time worked, namely

$$L_1 > L_2 \quad \text{and} \quad l_1 > l_2. \tag{16}$$

However,

$$L_1 = K - l_1 \quad \text{and} \quad L_2 = K - l_2,$$

so that

$$L_1 > L_2 \Rightarrow l_2 > l_1. \tag{17}$$

How to reconcile (16) and (17)? The former would imply the opposite inequality to the latter. However, when the externalities are taken into account, the explanation is simple. In the next section we shall show that the externality that the home produces on the firm, namely the parameters γ_1 and γ_2, do reconcile these two apparently divergent inequalities.

9. Externalities: Inequity at Home Reduces Women's Productivity in the Market

As already pointed out, the amount of work that a person performs at home has an impact on their productivity in the marketplace. The first hour that a woman works at the firm may be the 6th hour of work that day, since she may have worked already 5 hours at home.

Yet the number of hours that a person works at home are not known to the firm, nor can the firm control them. This is an *externality* that the family causes the firm. Formally,

l_1 and l_2 are treated as parameters by the firm even though they have an impact on the firm through worker's productivity. These observations may be formalized as follows.

ASSUMPTION 2. *There exists a parameter $\gamma_i > 0$ representing an "externality" on the firm so that for $i = 1, 2$*

$$\frac{\partial f}{\partial L_i} = \frac{\partial f}{\partial L_i}(\gamma_i), \quad \text{where } \frac{\partial^2 f}{\partial \gamma_i \partial L_i} < 0.$$

A simple example of this phenomenon would be

$$f(L_i) = \gamma_i(l_i) L_i^\alpha,$$

where

$$\gamma_i = \gamma_i(l_i) \quad \text{and} \quad \partial \gamma_i / \partial l_i < 0.$$

Under Assumption 2 above:

PROPOSITION 3. *Inequity at home leads to lower productivity of women at work, and to lower salaries for women.*

PROOF. This is an immediate consequence of Assumption 2 and (13). □

The productivity of women in the marketplace depends on the amount of time they work at home. This breaks the symmetry between productivity at work and hours worked. Even if the production function f is concave, those who spend more time working at home could have lower productivity in the marketplace while working fewer hours than the rest. This is because the production function f depends not only on L but also on l and at higher levels of l the graph of $f(L)$ shifts downwards due to the externality. This resolves the apparent conflict in (16) and (17) above.

10. Inequity Lowers Family Welfare

We saw that inequity at work leads to inequity at home and that inequity at home reduces productivity at work for those working longer hours at home. If women are subject to this inequity, then obviously they are worse off under these conditions. Is it possible, however, that the family as a whole is better off? The following proposition provides a response.

PROPOSITION 4. *At higher levels of output, inequity lowers family welfare, decreasing both family services h and the family's consumption of market goods x.*

PROOF. We have already shown that, under these conditions, the family produces more home services h with the same total amount of labor if the work load is distributed equally between the two genders, i.e. each allocates $\frac{l_1 + l_2}{2}$. Namely, when $\frac{l_1 + l_2}{2} > \frac{\beta}{\gamma}$,

$$l_1 \neq l_2 \Rightarrow 2g\left(\frac{l_1 + l_2}{2}\right) > g(l_1) + g(l_2),$$

so that inequity leads to less family services h. Yet it is still possible that inequity at home could increase family income sufficiently to compensate for the loss in family services. We show that this is not possible under these conditions. By definition, inequity at home means $l_1 < l_2$

$$\text{which implies } L_1 = K - l_1 > L_2 = K - l_2,$$

This, under these conditions, implies that women's marginal productivity at work is lower than men's (see (13)). Since the firm has a logistic production function f then, for the same total amount of labor $L_1 + L_2$, an equal workload among women and men (each works italic l subscript 1 plus italic l subscript 2 all over 2) increases total output:

$$2f\left(\frac{L_1 + L_2}{2}\right) > f(L_1) + f(L_2) \quad \text{when } L_1 \neq L_2,$$

as shown in Proposition 1. Therefore the total production of market goods x is lower than when men and women share work equally. Since all production is consumed by families, the family consumes less market goods x, as well as fewer family services. Therefore inequity at home lowers the family's welfare. □

11. Inequity Leads to Lower Output and Lower Profits

PROPOSITION 5. *At higher output levels, inequity reduces the firm's output and lowers its profits.*

PROOF. We saw in Proposition 4 that under these conditions, inequities decrease the market's output of x. For the same total amount of work, the production of the firm is higher when men and women divide equally the work load:

$$2f\left(\frac{L_1 + L_2}{2}\right) > f(L_1) + f(L_2), \quad \text{when } L_1 \neq L_2.$$

This proves the first part of the proposition. It remains to consider the impact of inequity on profits, namely on the function

$$\pi(L_1, L_2) = f(L_1) + f(L_2) - w_1 L_1 - w_2 L_2,$$

We wish to compare

$$\pi(L_1) + \pi(L_2) \quad \text{with} \quad 2\pi\left(\frac{L_1 + L_2}{2}\right).$$

By concavity (since we are above the inflection point of f) profits increase with the level of output, namely

$$\frac{\partial \pi}{\partial x} > 0.$$

Since equity increases output, and profit is an increasing function of output, it follows that equity increases profits as well. Equivalently, inequity decreases output and profits. □

12. A Nash-Walrasian Solution

This section describes the functioning of the economy as a whole. The economy consists of a Walrasian market, where firms maximize profits, and of families that produce public goods using common property resources, maximizing welfare. There are three traded goods in the economy: the market good x, women's labor, and men's labor. We normalized the price of x so that $p_x = 1$.

Recall that the family is not Walrasian; its services h are shared among the members, which makes them similar to (local) public goods. Furthermore, the resources such as labor l_1 and l_2, that are used to produce h, are allocated by common decision within the family so as to maximize the family's welfare. Therefore the family treats resources as common property. Additionally the family produces an externality on the firm γ_i, which depends on the hours that men and women work at home, $\gamma_i = \gamma_i(l_i)$, $i = 1, 2$. There are no benchmark models to analyze the functioning of such a mixed economy. We need some definitions.

DEFINITION 1. *If* $w_1 \neq w_2$ *we say that* the market is unfair. *If* $w_1 = w_2$ *we say that* the market is fair.

DEFINITION 2. *If* $l_1 \neq l_2$ *we say that* the family is *unfair and if* $l_1 = l_2$ *we say that* the family is *fair*.

PROPOSITION 6. *Given wages for the two types of labor* w_1 *and* w_2 *from the family's welfare optimization behavior (3) it is possible to determine the amount of family services it produces, the employment of men's and women's labor at home,* l_1 *and* l_2*, the offer of labor of the two types to the marketplace,* $K - l_1$ *and* $K - l_2$*, the family's demand for market goods, the family's income, its welfare level, and the value of the externality parameters* $\gamma_1(l_1)$ *and* $\gamma_2(l_2)$ *which modify the firm's production function. On the other hand, the firm has expected values for the parameters* γ_1^e and γ_2^e *and from the firm's profit maximization behavior (1) it is possible to determine the amount of labor the firm wishes to employ (men and women), how much it produces, what are its profits, and the productivity of its labor.*

PROOF. This is a standard microeconomic exercise. □

In Proposition 6 the family and the firm may have contradictory goals in terms of the productivity parameters γ_1^e and γ_2^e, the market goods produced and consumed, and, people employed. A *solution* for this economy arises when firms and families behave consistently.

DEFINITION 3. *A Nash-Walrasian solution for this economy consists of wages for men and for women* w_1^*, w_2^* *and expected values of the parameters* γ_1^e, γ_2^e *leading to consistent behavior by the family and the firm. The levels of employment and consumption that derive from profit optimization by the firm and from welfare optimization by the family clear all three markets, and the value of the externality produced by the family on the firm equals the values expected by the firm.*

In particular a solution satisfies:

1. Expectations are confirmed

$$\gamma_i(l_1) = \gamma_1^e \quad \text{and} \quad \gamma_i(l_2) = \gamma_2^e.$$

2. Supply of men's labor equals demand for men's labor by the firm

$$L_1^D(w_1, w_2) = N. \arg\max \pi(w_1, w_2) = L_1^S(w_1, w_2) = 15 - l_1(w_1, w_2). \tag{18}$$

3. Supply of female labor equals demand of women's labor by the firm

$$L_2^D(w_1, w_2) = N. \arg\max \pi(w_1, w_2) = L_2^S(w_1, w_2) = 15 - l_2(w_1, w_2). \tag{19}$$

4. Supply by the firm of x equals the family's demand for x,

$$x^S(w_1, w_2) = f\left(L_1^D(w_1, w_2), L_2^D(w_1, w_2)\right) = x^D(w_1, w_2) = w_1 L_1 + w_2 L_2 + \pi. \tag{20}$$

The existence of a *Nash-Walrasian solution* (as defined above) shows that the model as postulated is internally consistent.

PROPOSITION 7. *There exists a solution for this economy.*

PROOF. *see Appendix.*

13. The Market-Family Game

This section defines a game with two players, the market and the family. *The market's objective* is to maximize profits as defined in (1). The *family's objective* is to maximize welfare as defined in (3). The players choose their strategies to achieve their goals. The *market's strategy* is to set wages for men and for women, w_1 and w_2, and expectations about their productivity γ_1^e and γ_2^e while the *family's strategy* is to allocate labor at home among men and women, l_1 and l_2.

DEFINITION 4. ***A Nash equilibrium** is a set of strategies for the market and for the family* $(w_1^*, w_2^*, \gamma_1^e, \gamma_2^e, l_1^*, l_2^*)$ *leading to a solution for the economy in which each player reacts optimally to the other's strategy, and neither has an incentive to deviate.*

PROPOSITION 8. *At high levels of output:*

1. *There is a Nash equilibrium where women have lower salaries than men, and the family reacts by allocates more housework to women. Conversely, at a Nash equilibrium where the family allocates more housework to women, women productivity is lower in the marketplace and they receive lower salaries than men. This Nash equilibrium is called* unfair–unfair.
2. *There is a Nash equilibrium where women have the same salaries as men. Women have the same marginal productivity. The family reacts by sharing equally housework between men and women. Conversely, at a Nash equilibrium where women and men share housework equally, their wages in the marketplace are the same as men's. This is a* fair–fair *Nash equilibrium.*
3. *The* unfair–unfair *Nash equilibrium is Pareto inferior. The* fair–fair *Nash equilibrium is Pareto efficient, but it is riskier.*

Proof. When women have the same salaries as men, both bring the family the same income for the same hours in the marketplace. By (13) their productivity is the same at an optimum, and, given the assumptions, it is more productive for both men and women to work the same hours in the markeplace. At the same time, by Proposition 1, women work at home the same number of hours as men, since under the conditions, sharing work equally at home provides more family services for the same total amount of labor.

Reciprocally, when women and men share work equally at home, then it is optimal for the firm to pay both equally from (13). The *fair–fair* pair of strategies just described is a Nash equilibrium of the market–family game because when following such a pair of strategies, each player responds optimally to the other's move.

At a Nash equilibrium, where women's salaries are inferior to men's, it is optimal for the family to choose an unfair distribution of household work, by Proposition 2. Women work more at home, and their productivity at home is lower, as shown in Proposition 2 and in section 10, and so is their productivity at work, by (13). This is an *unfair–unfair* Nash equilibrium, with both players responding optimally to each other, but nevertheless, it is a Pareto inferior solution.

The *first fair–fair* equilibrium is Pareto optimal. The following section illustrates why the fair–fair equilibrium is riskier under these conditions. □

14. A Matrix Game

The matrix below illustrates a game where the horizontal strategies represent the market's and the vertical represent the family's. The pay-offs for the market are sub-indexed 1 and those for the family are sub-indexed 2.

$$\begin{array}{ccc} & w_1 \neq w_2 & w_1 = w_2 \\ l_1 \neq l_2 & (A_1, A_2) & (C_1, D_2). \\ l_1 = l_2 & (D_1, C_2) & (B_1, B_2) \end{array}$$

In this matrix game, Proposition 8 above can be illustrated by the inequalities

$$C_1 < A_1 < B_1 < D_1,$$

and

$$C_2 < A_2 < B_2 < D_2,$$

where (A_1, A_2) is the outcome of the unfair–unfair Nash equilibrium, (B_1, B_2) is the outcome of the fair–fair Nash equilibrium. The fair–fair Nash equilibrium is Pareto efficient because $A_1 < B_1$ and $A_2 < B_2$.

The Pareto efficient Nash equilibrium is more risky, because $C_1 < A_1$, so if the market plays *fair* but the family plays *unfair* the market will be worse off. This is Proposition 3. Conversely, $C_2 < A_2$ implies that the family will be worse off if it plays *fair* while the market plays *unfair*. This by Proposition 2.

15. The Family-Market Game Is Similar to the Prisoner's Dilemma

The matrix presented above is similar to that of the "prisoner's dilemma game" when, in addition to the inequalities,

$$C_1 < A_1 < B_1 < D_1,$$

and

$$C_2 < A_2 < B_2 < D_2,$$

the two players are symmetrically situated, so that

$$A_1 = A_2, \quad B_1 = B_2, \quad C_1 = C_2, \quad D_1 = D_2.$$

A numerical example of the prisoner's dilemma is

$$\begin{pmatrix} 5, 5 & 3, 10 \\ 10, 3 & 9, 9 \end{pmatrix},$$

while a numerical example of our situation need not be symmetrical—for example

$$\begin{pmatrix} 5, 6 & 3, 10 \\ 9, 4 & 8, 9 \end{pmatrix},$$

where

$$\begin{aligned} A_1 &= 5 \quad A_2 = 6 \\ B_1 &= 8 \quad B_2 = 9 \\ C_1 &= 3 \quad C_2 = 4 \\ D_1 &= 9 \quad D_2 = 10. \end{aligned}$$

16. Conclusions

The coupling of two distinct institutions—the market and the family—can lead to a disproportionate allocation of home responsibilities to women, and simultaneously to the lowering of women's wages. We showed that there is a cooperative solution that is better for all, involving equity at home and in the workplace. However, this solution seems riskier because if either the family or the firm deviate from it, the other is worse off. The family loses if it plays fair when the market doesn't, and vice versa. (See Engineer and Welling, 1999; Edin and Richardson, 2002; and Elul et al., 2002.) The risks can be seen to derive from missing contracts between the family and the marketplace.

What social institutions can help resolve this problem? Waldfogel (1998) and others have considered similar issues. A prenuptial agreement that specifies women's and men's roles in the family could be a start. It should have penalties attached if the parties default from what was promised. Using such a legal agreement, women can present themselves at work as fully able to deliver, so a fair employer is not misled about the nature of the labor it hires.

Similarly, strengthening equal pay provisions in the marketplace should support the execution of these prenuptial agreements. This requires enforcing the Equal Pay Act—and perhaps making this enforcement contingent on the availability of the prenuptial agreement just discussed. This way the firms would not risk being penalized for playing fair.

Other solutions to the prisoner's dilemma have been proposed over the years; most of them encourage cooperation among the players. Often this requires repeated games among the players, which is not realistic in the case of marriage (Lagerlöf, 2003). In any case, any solution that encourages a cooperative outcome between the family and the market will benefit both. The moral of this article is that equity may appear to be riskier—and, indeed, it may be—but it is after all the Pareto-efficient allocation. Room should be made for the missing contracts between the players—the market and the family—that take advantage of the existence of a win–win solution, making everyone better off.

Appendix

Proof of *Proposition 7*

We show the existence of a *solution* (as defined above) in a simple case; the most general case requires the use of a fixed point argument. The simplest (nontrivial) case is when $\frac{w_1}{w_2} > M$ as defined in (14). Under these conditions, as we saw in Proposition 2, women will do all the housework and men will only work in the marketplace. From (13), we obtain the total amount of hours that women work at home, denoted l_2, which, as already discussed, produces an externality on the productivity of women at the firm. There is no externality in the case of men, because men do not work at home. Therefore the total amount of hours that men work at the firm is L_1 as determined from (13) and so is the marginal productivity $\frac{\partial f}{\partial L_2}$. Because we know the ratio of wages $\frac{w_1}{w_2}$ from (13), we may now derive the number of hours L_2 that women work at the firm, together with the value of the externality γ_2—the two values L_2 and γ_2 must satisfy the equations

$$\frac{w_1}{w_2} = \frac{\frac{\partial f}{\partial L_1}(\gamma_1)}{\frac{\partial f}{\partial L_2}(\gamma_2)}, \tag{21}$$

and

$$K - L_2 = l_2. \tag{22}$$

To solve the model we need to find the values of the variables, γ^* and L_2^*that satisfy the two equations (21) and (22). One shifts the production function using the externality parameter γ_2 until the two equations are satisfied this provides a solution for γ_2. At a solution, the productivity of women at the firm will be lower than men's, because women work most of their time at home. The vector $(w_1^*, w_2^*, \gamma_2^*, l_1^*, l_2^*)$ is a solution for this economy. □

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Notes

1. The problem persists across all occupations and income levels, and is typically worse at the top. See Rosholm and Smith (1996), Meyersson Milgrom et al. (2001), Bureau of Labor Statistics (2003), Gupta et al. (2003), Meyersson Milgrom and Petersen (2003), Blau and Kahn (2004), Ginther (2004), NCES (2004).
2. In economic terms, there are *externalities* between the market and the family because the more a person works at home, the less reliable or productive they can be in the marketplace. In legal terms, there are missing *property rights* and missing contracts between the two institutions. Both of these issues impede the work of the market; they tie down the invisible hand.
3. Health services is an important sector, representing about 16% of the US GDP.

4. Coltrane (2000) shows that women spend 2–3 times as many hours in routine houeswork as men. Davis, Greenstein and Marks (2007) use data from 17,636 respondants in 28 nations and find that relative financial contributions area a key determinant of household division of labour.
5. To clarify this issue the experiments of Gneezy et al. should have been augmented to ask the women and the men who participated the amount of time they spent in each of the two institutions. In the case of students, the question may have been better posed in terms of the amount of time they expected to spend in each of the two institutions—or the amount of time that their "gender role models"—such as parents or teachers—themselves spend at home and in the marketplace.
6. Holmstrom and Milgrom (1991) examine people who share their time among different activities and predict specialization, as does Becker. Their production functions have increasing productivity, and as a result each task is the responsibility of a single person, thus predicting hierarchies. Under our conditions, instead, we show that at higher levels of employment equal sharing at home and at the marketplace emerges as the more productive strategy. This increases family welfare, and is more productive in the workplace.
7. Davis, Greenstein and Marks (2007) confirm this empirically: "Consistent with previous research, we find more egalitarian division of labor (at home) in countries with greater gender equity" (p. 1266).