

an education she would not be able to use to the full. By limiting parental freedom to choose the mix of money and education to give a child, compulsory education makes it less likely that a girl will have a cooperative marriage, and more likely that it will be characterized by equal sharing of child care and market work. All of this assumes that education gives utility only indirectly, by raising the recipient's earning capacity. If education gives also direct utility, or has a direct effect on a person's domestic bargaining power, the argument for giving a daughter money rather than education becomes weaker.

Notes

- 1 Each of them would then consume $(1 + \alpha) h\omega$ of the private good, and zero of the public one. Utility would be $u((1 + \alpha) h\omega)$ for both of them.
- 2 In that limiting case, however, there would be nothing to gain from domestic division of labour.

References

- Basu, K. (2006) 'Gender and Say: A Model of Household Behaviour with Endogenously Determined Balance of Power', *Economic Journal*, 116: 558-580.
- Bergström, T. C. (1996) 'Economics in a Family Way', *Journal of Economic Literature*, 24: 1903-1934.
- Cigno, A. (1991) *Economics of the Family*, Oxford and New York: Clarendon Press and Oxford University Press.
- Cigno, A. (2006) 'A Constitutional Theory of the Family', *Journal of Population Economics*, 19 (Special Issue on Political Economy and Population Economics): 259-283.
- Cigno, A. and Rosati, F. C. (2005) *The Economics of Child Labour*, Oxford and New York: Oxford University Press.
- Lundberg, S. and Pollak, R. A. (1996) 'Bargaining and Distribution in Marriage', *Journal of Economic Perspectives*, 10: 139-158.
- Lundberg, S. and Pollak, R. A. (2003) 'Efficiency in Marriage', *Review of Economics of the Household*, 1: 153-167.
- McElroy, M. B. and Horney, M. J. (1981) 'Nash-Bargained Household Decisions', *International Economic Review*, 22: 333-349.
- Manser, M. and Brown, M. (1980) 'Marriage and Household Decision Making: a Bargaining Analysis', *International Economic Review*, 21: 31-44.

4 The gender gap

*Graciela Chichilnisky*¹

Introduction

The gender gap, like the minority achievement gap, has lately become a hot topic. Women are underpaid, undervalued, and overworked across the board. But in our rational economy, what could explain the persistence of this phenomenon? A preferential demand for lower paid women should drive their salaries up until they reach the level of men's. The logic seems impeccable, but it is not borne out by the facts.² This article provides an explanation based on the coupling of two institutions: the family and the market. Families are about sharing and using common property resources. Firms, instead, use private property to produce private goods, and maximize profits. As far as institutions go, the family and the market could not be further apart, yet they are undeniably intertwined. The way that each responds to the other is critical in understanding and resolving the unequal situation of women in our society.

I hope to explain the seemingly illogical actions of the family-market system by introducing a game between the two components. This game helps to explain the gender gap in salaries, and why men and women allocate time differently between work and home. I show that inequality at work leads to inequality at home, and vice versa. This vicious circle creates a persistent gender gap. The government may regulate the workplace, but it cannot regulate the family. Since one inequality cannot be solved without the other, this may explain why the gender gap has been so difficult to overcome.

The current situation has evolved over time. Women had lower salaries historically, and therefore performed most housework because men could make a higher income working in the marketplace. Under the conditions, the traditional division of labour is a rational way to maximize family income. However, the burden of excessive housework decreases the time and the energy that women can bring to the marketplace. Therefore the family produces externalities on the firm.³ My point is that under these conditions a firm will perceive women as being more risky than men because they are not available in case of emergencies and ill health, since, for example they are the main providers of the medical needs of the family, or are less productive than men, since they have demanding second jobs at home that men do not have.⁴ If workers are assets, then women are riskier assets even

when they are equally productive. This riskiness is in turn used to justify women's lower wages, closing the vicious circle. From this game between the family and the marketplace, the gender gap emerges as a rational but undesirable situation that is similar to the classical prisoner's dilemma.

The perception that women are more risky workers is felt most acutely in the most demanding and highest paid jobs where constant availability is required. This means that at the highest levels, there should be a larger gap in female participation and salaries. This could be an explanation for the glass ceiling, the somewhat perverse phenomenon that leads the more productive women to face higher differentials between their compensations and men's (Meyersson Milgrom *et al.* 2001; Meyersson Milgrom and Petersen 2003).

The empirical and experimental evidence appears to confirm the above observations. Bonke *et al.* (2005) found that larger differences between men and women's work at home are associated with larger differences in market salaries, and recent experiments by Gneezy *et al.* (2003) show that women perform worse than men in competitive environments. Both make sense. Due to their lower salaries, women spend more time working at home where the most important skills involve sharing and cooperation. One can therefore expect women to adapt to the cooperative family 'mores', while men adapt instead to the competitive 'mores' of the marketplace. After all, success in the marketplace requires competitive skills – while success at home requires instead cooperative skills.⁵ Recent work validates the empirical conclusions reached in this chapter (Chichilnisky and Shachmurove 2007).

This article formalizes a toy game where women and men share their time between the family and a Walrasian market economy. They learn by doing, in the sense that the more they work, the more productive they are. This follows Becker's classic article (Becker 1985), which provides the standard argument for specialization of women and men, at home or in the marketplace respectively.⁶ In contrast with Becker's assumption, however, I follow Arrow's 1962 seminal article where he introduced learning by doing. In Arrow's formulation there are decreasing returns after a certain number of hours per day (Arrow 1962). I show that Arrow's model reverses Becker's findings in the sense that specialization is no longer necessary for efficiency at higher levels of productivity. There is now another, rational, solution in which women and men are paid the same and share the work equally in both institutions. This fair outcome emerges at higher levels of output, when the economy is richer and more productive. Once production exceeds a minimum level we enter Arrow's regime where the learning curve is concave rather than convex as in Becker's work. I show that a new equilibrium emerges that leads to more welfare at home, more family services, and simultaneously to higher productivity and profits in the marketplace. Inequality is no longer the only solution. Now fairness is Pareto efficient.

If such equitable solutions exist, one may ask, why aren't they observed more often? The answer is that under current economic and social conditions, the equitable solution seems riskier, as is the optimal solution in the prisoner's dilemma. It can be seen that there are missing contracts between the players. Equal treatment in the family depends on equal treatment in the marketplace, and

vice versa – but neither institution can safely depend on the other, and they do not have contracts to implement the optimal solution. In the conclusions I suggest how certain incentives or firm informational structures can help overcome the problem, and even the introduction of new contractual arrangements that can help overcome the lack of contracts between the parties and help reach efficient and equitable social solutions.

The firm

The economy has several identical competitive firms producing a good x . A representative firm uses two types of workers, men and women. Their labour is denoted L_1 and L_2 respectively with possibly different wages w_1 and w_2 . The firm's production technology is described by a function f

$$x = f(L_1) + f(L_2)$$

The firm's goal is to maximize profits π , namely the difference between the firm's revenues and its costs:

$$\text{Max}_{L_1, L_2}(\pi) = \text{Max}_{L_1, L_2} [p_x(f(L_1) + f(L_2)) - (w_1 L_1 + w_2 L_2)] \quad (4.1)$$

Since firms are competitive they take the price of goods x , p_x , and wages w_1 and w_2 , as parametrically given. Maximizing profits implies the standard condition that wages must equal the marginal product of labour:

$$w_1 = \frac{\partial f}{\partial L_1} \text{ and } w_2 = \frac{\partial f}{\partial L_2} \quad (4.2)$$

In the following I assume that there are two parameters γ_1 and γ_2 which vary with the person's work at home and influence their productivity in the marketplace. The firm takes these parameters as given; they represent an externality⁷:

$$x = f(L_1, \gamma_1) + f(L_2, \gamma_2)$$

so for each given γ_1, γ_2 profit maximization implies

$$w_1 = \frac{\partial f}{\partial L_1}(\gamma_1) \text{ and } w_2 = \frac{\partial f}{\partial L_2}(\gamma_2)$$

The family

There are several identical families. Neglecting distributional issues we refer to a 'representative' family whose welfare derives from family services h , and from the consumption of goods x . The family goal is to optimize welfare:

$$\text{Max}(U(x, h)) \quad (4.3)$$

Family services are produced according to a technology g

$$h = g(l_1) + g(l_2) \quad (4.4)$$

where l_1 and l_2 are the two types of labour in the household, men's and women's respectively. Let K be the total amount of hours that a person can feasibly work in a given period of time, at home and in the market. As an example, in a given day, this could be $K = 15$. When all labour is utilized

$$L_1 = K - l_1 \text{ and } L_2 = K - l_2 \quad (4.5)$$

The family's income equals the wages that its members, a man and a woman, earn in the marketplace plus the firms' profits, since families own the firms. The value of what the family buys $p_x x$ must equal its income:

$$p_x x = w_1 L_1 + w_2 L_2 + \pi \quad (4.6)$$

where as before profits π are the firm's revenues minus its costs:

$$\pi = p_x (f(L_1, \gamma) + f(L_2, \gamma)) - (w_1 L_1 + w_2 L_2) \quad (4.7)$$

We normalize by assuming that the price of x is one, $p_x = 1$, so that the family's 'budget' equation is

$$x = f(L_1, \gamma) + f(L_2, \gamma) \quad (4.8)$$

The family's trade-off

The family faces a trade-off in deciding whether to use labour at home or in the marketplace. The more labour is used at home, the more family services are produced, but the lower is the family's income and therefore the fewer market goods it consumes. The family has to reach an optimal use of labour at home and in the marketplace to optimize its welfare.

When women and men are paid differently, $w_1 \neq w_2$, the family's decision problem by (4.5), (4.4), and (4.8) is to choose l_1, l_2 to

$$\text{Max}_{l_1, l_2} U(f(K - l_1, \gamma) + f(K - l_2, \gamma), g(l_1) + g(l_2)) \quad (4.9)$$

The family considers the productivity parameters γ_1 and γ_2 as given.⁸ From (4.2) this implies

$$\frac{\partial U}{\partial x}(-w_1) + \frac{\partial U}{\partial h} \frac{\partial g}{\partial l_1} = 0 \quad (4.10)$$

and

$$\frac{\partial U}{\partial x}(-w_2) + \frac{\partial U}{\partial h} \frac{\partial g}{\partial l_2} = 0$$

Therefore wages determine the productivity of each type of labour at home, and the amount of time each works at home

$$\frac{\partial g}{\partial l_1} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} w_1 \text{ or } w_1 = \frac{\partial g}{\partial l_1} \frac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial x}} \quad (4.11)$$

$$\frac{\partial g}{\partial l_2} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial h}} w_2 \text{ or } w_2 = \frac{\partial g}{\partial l_2} \frac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial x}} \quad (4.12)$$

Equivalently, we obtain the standard result that the marginal rate of substitution between home services and market goods equals their marginal rates of transformation, which in turn equal the ratio of wages:

$$\frac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial x}} = \frac{\frac{\partial g}{\partial l_1}}{\frac{\partial g}{\partial l_2}} = \frac{\frac{\partial f}{\partial L_1}}{\frac{\partial f}{\partial L_2}} = \frac{w_1}{w_2} \quad (4.13)$$

Public goods and common property resources

We may consider a family that acts as a single unit, making choices about how to allocate women and men's labour, namely l_1 and l_2 . This means that the family's labour is treated as common property. Furthermore, since there is a single welfare level for the entire family, this means that family services are shared as a 'public good' within the family (see also Apps and Rees 1997; Aronsson *et al.* 2001).

This is summarized by saying that the family produces a public good using common property resources. Family services are better described as a 'local' public good within the family, because they are not shared with other families.

Learning by doing

Becker pointed out that the more time we spend in a given activity the better we become at doing it (Becker 1985). This is called *learning by doing*. It means that marginal productivity \dot{g} increases with time. Under these conditions, each person in the family (man or woman) should specialize – one should specialize in working at home, and the other in the marketplace. Both are more productive, at home and in the marketplace, thus increasing family welfare. As a direct consequence of Becker's assumption, when women's salaries are lower than men, women should do all the housework. Men should only work in the marketplace.

Since in fact women's salaries are lower than men's in most economies, both historically and currently, Becker's assumption leads directly to a division of labour

where women stay at home and men work in the marketplace. Under Becker's assumptions the current situation where most household work is done by women seems a rational and efficient solution.

There is indeed learning by doing in our society and therefore Becker's assumption is reasonable, but only up to a point. Human beings need rest after a number of working hours, and this implies a decrease in productivity beyond a certain number of hours of work.

Accordingly, we assume here that the time derivative of the home production function \dot{g} is initially positive, but after a maximum is reached \dot{g} starts to decrease since humans cannot work productively without rest.

If $g(t)$ is the amount of h produced with t hours worked, then we may assume that increases in productivity follow a modified quadratic form, increasing initially and then decreasing as was just postulated,

$$\dot{g}_t = H(g_t) = \beta g - \gamma g^2 \text{ with } \beta, \gamma > 0$$

This equation integrates to yield the classic logistic curve that is used often to describe the evolution of biological populations over time:

$$g(t) = \frac{\beta g_0}{\gamma z_0 + (\beta - \gamma z_0) \exp(-\beta t)}$$

The logistic function $g(t)$ has an inflection point: e.g. when $g_0 = 1$, the inflection point is at $g = \frac{\beta}{\gamma}$. Assuming that $g_0 \approx 1$, the evolution over time of labour productivity increases with the number of hours worked, until it reaches a maximum increase at $g = \frac{\beta}{\gamma}$ and declines afterwards. The second derivative is positive until the inflection point and negative afterwards. The graph of the function is therefore convex until the value $\frac{\beta}{\gamma}$ and it is concave thereafter.

The convex part is similar to Becker's assumption and yields similar results. On the other hand the concave part, which occurs after the inflection point is reached yields very different results as shown further. The inflection point determines a change from one regime to the other; it appears in the diagram as the maximum of the quadratic curve, which is the derivative of g (Figure 4.1).

Assumption 1. In the following we assume that production has reached the inflection point at home and at the marketplace, an assumption that seems to make sense in highly productive economies. We describe this as having achieved higher levels of output.

Equity at home improves welfare

Proposition 1. At higher levels of output, equity benefits the family. In other words, distributing home labour equally between men and women produces more

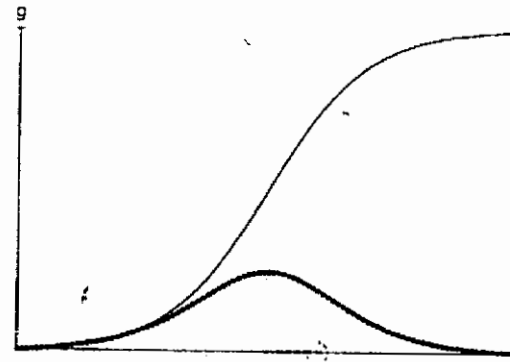


Figure 4.1 Logistic function $g(t)$.

household services for the same total labour. Formally, if

$$\frac{l_1 + l_2}{2} > \frac{\beta}{\gamma}$$

where β/γ is the inflection point of the logistic, then

$$l_1 \neq l_2 \Rightarrow g(l_1) + g(l_2) < 2g\left(\frac{l_1 + l_2}{2}\right)$$

Proof:

$$2g\left(\frac{l_1 + l_2}{2}\right) > g(l_1) + g(l_2) \Leftrightarrow g\left(\frac{l_1 + l_2}{2}\right) > \frac{g(l_1)}{2} + \frac{g(l_2)}{2}$$

which is implied by the definition of concavity. Recall that above its inflection point the logistic curve g is concave, since its second derivative is negative. This proves the inequality. In words: equity is a more efficient use of resources at home whenever

$$\frac{l_1 + l_2}{2} > \frac{\beta}{\gamma}$$

as we wished to prove.

Inequality at work leads to inequality at home

There is historic difference in the average pay of men and women, about 25 or 30 per cent in the US. What is the optimal response by the family to this inequality,

in terms of allocating labour at home? The following proposition provides a response:

Proposition 2. Inequality at work leads to inequality at home when women are paid less than men in the marketplace, $w_1 > w_2$, the family's optimal response is that women should work longer hours at home than men. When the difference in wages is large enough, $\frac{w_1}{w_2} > M = \frac{\sup \frac{\partial g}{\partial l_1}}{\inf \frac{\partial g}{\partial l_2}}$, it is optimal for the family that women should do all the housework, and men should work only in the marketplace.

Proof: Proof: From (4.3) and (4.9) the family's goal is

$$\text{Max}_{l_1, l_2} U(f(K - l_1) + f(K - l_2), g(l_1) + g(l_2))$$

From (4.13)

$$\frac{\frac{\partial g}{\partial l_1}}{\frac{\partial g}{\partial l_2}} = \frac{w_1}{w_2}$$

so that at an optimum

$$w_1 > w_2 \text{ implies } \frac{\partial g}{\partial l_1} > \frac{\partial g}{\partial l_2}$$

Therefore women (over)work at home, in the sense that they work up to the point where their marginal productivity is lower than men's.⁹ As we saw in the section learning by doing, when $g(t) > \frac{t}{\gamma}$, the marginal productivity of labour $\frac{\partial g}{\partial t}$ is a decreasing function of the time allocated, so that lower productivity means longer hours for women at home.

When the ratio of salaries exceeds M , the ratio of the supremum and the infimum productivity of g , namely when

$$\frac{w_1}{w_2} > M = \frac{\sup \frac{\partial g}{\partial l_1}}{\inf \frac{\partial g}{\partial l_2}} \quad (4.14)$$

it is optimal that women should completely specialize in housework. This completes the proof.

Proposition 2 implies that it is always optimal for the family to use more female labour at home when they have lower salaries than men. If women's housework hours are less than the maximum feasible, K , then it would be rational that women should also work in the marketplace in addition to their work at home – at their reduced salaries. Furthermore, when salary differentials are large enough, it is

optimal for the family that women do all the housework and that they work also in the marketplace receiving lower salaries than men, while men, on the other hand, work only in the marketplace and receive higher salaries.

The logic of the situation and (4.13) imply that when $w_1 > w_2$, then women's marginal productivity is lower than men's at home and also in the marketplace. When production functions f and g are concave, this implies in turn that women work more hours than men at home and also in the marketplace, because marginal productivity decreases with the time worked, so that:

$$L_1 > L_2 \text{ and } l_1 > l_2 \quad (4.15)$$

However on the other hand,

$$L_1 = K - l_1 \text{ and } L_2 = K - l_2$$

so that

$$L_1 > L_2 \Rightarrow l_2 > l_1 \quad (4.16)$$

How should we reconcile the apparent contradiction between (4.15) and (4.16)? In the next section we show that the externality that the home produces on the firm, namely the parameter γ , reconcile these two apparently contradictory inequalities.

Externalities: inequality at home reduces women's productivity in the market

As already pointed out, the amount of work that a person performs at home has an impact on their productivity in the marketplace. The first hour that a woman works at the firm may be the sixth hour of work that day, since she may have already worked five hours at home.

Yet in a competitive market, the number of hours that a person works at home before going to work at the firm is private information and not known to the firm, nor can the firm control them. This is an externality that the family causes the firm. Formally, l_1 and l_2 are treated as parameters by the firm even though they have an impact on the firm through worker's productivity. These observations may be formalized as follows:

Assumption 2. There exists a parameter $\gamma > 0$ representing an 'externality' on the firm so that

$$\text{for } i = 1, 2 \quad \frac{\partial f}{\partial L_i} = \frac{\partial f}{\partial L_i}(\gamma) \text{ where } \frac{\partial^2 f}{\partial \gamma \partial L_i} < 0.$$

A simple example of this phenomenon would be

$$f(L_i) = \gamma(l_i)L_i^\alpha$$

where

$$\gamma = \gamma(l_i) \text{ and } \partial\gamma/\partial l_i < 0.$$

Under assumption 2.

Proposition 3. Inequality at home leads to lower productivity of women at work, and to lower salaries for women. This is an immediate consequence of assumption 2 and (4.13).

The interpretation of the result is simple: as we already discussed, the productivity of women in the marketplace depends on the amount of time they work at home. This breaks the symmetry between productivity at work and hours worked, because even if the production function f is concave, those who spend more time working at home have a lower productivity in the marketplace while working fewer hours than the rest. Recall that the production function f depends not only on L but also on l , and at higher levels of l the graph of $f(L)$ shifts downwards due to the externality, i.e.

$$f(l) = f(L, \gamma) \text{ with } \partial f/\partial \gamma < 0.$$

The externality thus resolves the apparent contradiction between (4.15) and (4.16).

Inequality lowers family welfare

We saw that inequality at work leads to inequality at home and that inequality at home reduce productivity at work for those working longer hours at home. If women are subject to this inequality, then obviously they are worse off under these conditions. Is it possible however that the entire family is better off as a whole? The following proposition provides a response.

Proposition 4. At higher levels of output, inequality lowers family welfare, decreasing both family services h and the family's consumption of market goods x .

Proof: We have already shown that, under the conditions, the family produces more home services h with the same total amount of labour if the work load is distributed equally between the two genders. Namely when $\frac{l_1+l_2}{2} > \frac{g}{\gamma}$

$$l_1 \neq l_2 \Rightarrow 2g\left(\frac{l_1+l_2}{2}\right) > g(l_1)+g(l_2)$$

Therefore inequality leads to less family services h .

Yet it is still possible that inequality at home could increase family income sufficiently to compensate for the loss in family services. We show that this is not

possible under the conditions. By definition, inequality at home means $l_1 < l_2$ which implies

$$L_1 = K - l_1 > L_2 = K - l_2$$

This under the conditions implies that women's marginal productivity at work is lower than men's, see (4.13). Since the firm has a logistic production function f then for the same total amount of labour $L_1 + L_2$ an equal workload among women and men increases total output:

$$2f\left(\frac{L_1+L_2}{2}\right) > f(L_1)+f(L_2) \text{ when } L_1 \neq L_2$$

as shown in proposition 1. Therefore the total production of market goods x is lower than when men and women share work equally. Since all production is consumed by families, the family consumes less market goods x as well as fewer family services. Therefore inequality at home lowers the family's welfare as we wished to prove.

Inequality leads to lower output and lower profit

Proposition 5. At higher output levels, inequality reduces the firm's output and lowers its profits.

Proof: We saw in proposition 4 that under the conditions, inequities decrease the market's output of x . For the same total amount of work the production of the firms is higher when men and women divide equally the work load:

$$2f\left(\frac{L_1+L_2}{2}\right) > f(L_1)+f(L_2) \text{ when } L_1 \neq L_2,$$

This proves the first part of the proposition. It remains to consider the impact of inequality on profits, namely on the function

$$\pi(l_1, l_2) = f(L_1) + f(L_2) - w_1 l_1 - w_2 l_2$$

We wish to compare

$$\pi(L_1) + \pi(L_2) \text{ with } 2\pi\left(\frac{L_1+L_2}{2}\right)$$

By concavity (since we are above the inflection point of f) profits increase with the level of output, namely

$$\frac{\partial \pi}{\partial x} > 0$$

Since equity increases output, and profit is an increasing function of output, it follows that equity increases profits as well. Equivalently, inequality decreases output and profits as we wished to prove.

A mixed economy and a Nash-Walrasian solution

This section describes the functioning of the economy as a whole. The economy consists of a Walrasian market where firms maximize profits, and of families that produce public goods using common property resources, maximizing welfare. There are three traded goods in the economy: the market goods x , women's labour, and men's labour.¹⁰

This economy is partly Walrasian, and partly based on a common property resource that is used to produce a public good. There are no benchmarks for studying such a mixed economy. Indeed, the family is not Walrasian; its services h are shared among the members, which make them similar to (local) public goods. Furthermore, the resources such as labour l_1 and l_2 that are used to produce h are allocated by common decision within the family so as to maximize the family's welfare. Therefore the family treats resources as common property. Additionally the family produces an externality on the firm γ which depends on the hours that men and women work at home, $\gamma = \gamma(l_i)$, $i = 1, 2$. Since there are no benchmark models to analyze the functioning of such a mixed economy, we will propose a natural solution that is partly a Nash equilibrium and partly a Walrasian equilibrium, interacting with each other. For this we need some definitions.

- If $w_1 \neq w_2$ we say that *the market is unfair*. If $w_1 = w_2$ we say that *the market is fair*.
- If $l_1 \neq l_2$ we say that *the family is unfair* and if $l_1 = l_2$ we say that *the family is fair*.

Proposition 6. Finding a solution for the mixed economy. Given wages for the two types of labour w_1 and w_2 from the family's welfare optimization behaviour (4.3) it is possible to determine the amount of family services it produces, the employment of men and women's labour at home, l_1 and l_2 , the offer of labour of the two types to the marketplace, $K - l_1$ and $K - l_2$, the family's demand for market goods, the family's income, its welfare level, and the value of the externality parameters $\gamma_1(l_1)$ and $\gamma_2(l_2)$ which modify the firm's production function. On the other hand, the firm has expected values for the parameters γ_1^e and γ_2^e and from the firm's profit maximization behaviour (4.1) it is possible to determine the amount of labour the firm wishes to employ (men and women), how much it produces, what are its profits, and the productivity of its labour. The rest is a standard microeconomic exercise.

In proposition 6 the family and the firm may have contradictory goals in terms of the productivity parameters γ_1^e and γ_2^e , the market goods produced and consumed,

and people employed. A solution for this economy arises when firms and families behave consistently:

Definition. A solution for this mixed economy consists of wages for men and for women w_1^* , w_2^* and expected values of the parameters γ_1^e , γ_2^e leading to consistent behaviour by the family and the firm. The levels of employment and consumption that derive from profit optimization by the firm and from welfare optimization by the family clear all three markets. Furthermore, the value of the externality produced by the family on the firm equals the values expected by the firm.

In particular:

- (1) Expectations are confirmed;

$$\gamma(l_1) = \gamma_1^e \text{ and } \gamma(l_2) = \gamma_2^e$$

- (2) Supply of men's labour equals demand for men's labour by the firm;

$$L_1^D(w_1, w_2) = N \cdot \arg \max \pi(w_1, w_2) = L_1^S(w_1, w_2) = K - l_1(w_1, w_2) \quad (4.17)$$

- (3) Supply of female labour equals demand of women's labour by the firm;

$$L_2^D(w_1, w_2) = N \cdot \arg \max \pi(w_1, w_2) = L_2^S(w_1, w_2) = K - l_2(w_1, w_2) \quad (4.18)$$

and

- (4) Supply by the firm of x equals the family's demand for x ,

$$\begin{aligned} x^S(w_1, w_2) &= f(L_1^D(w_1, w_2), L_2^D(w_1, w_2)) = x^D(w_1, w_2) \\ &= w_1 L_1 + w_2 L_2 + \pi \end{aligned} \quad (4.19)$$

The existence of a solution shows that the model as postulated is internally consistent.

Proposition 7. There exists a solution for this mixed economy as defined above. See Appendix for the proof.

A particular example: the market – family game

This section illustrates the mixed economy with a game with two players, the market and the family. The market's objective is to maximize profits as defined in (4.1). The family's objective is to maximize welfare as defined in (4.3). The players choose their strategies to achieve their goals. The market's strategy is to set wages

for men and for women, w_1 and w_2 , and expectations about their productivity γ_1^e and γ_2^e , while the family's strategy is to allocate labour at home among men and women, l_1 and l_2 .

Definition. A Nash equilibrium for this game is a set of strategies for the market and for the family ($w_1^e, w_2^e, \gamma_1^e, \gamma_2^e, l_1^e, l_2^e$) leading to a solution for the economy in which each player reacts optimally to the other's strategy, and so neither has an incentive to deviate.

Proposition 8. At high levels of output:

- 1 Nash equilibrium where women have lower salaries. The family reacts by allocating more house work to women. Conversely, at a Nash equilibrium where the family allocates more housework to women, women productivity is lower in the marketplace and they receive lower salaries than men. This Nash equilibrium is called unfair-unfair.
- 2 Nash equilibrium where women have the same salaries as men. Women have the same productivity. The family reacts by sharing equally housework between men and women. Conversely, at a Nash equilibrium where women and men share housework equally, their wages in the marketplace are the same as men's. This is a fair-fair Nash equilibrium.

Theorem. The unfair-unfair Nash equilibrium is Pareto inferior. The fair-fair Nash equilibrium is Pareto efficient, but it is perceived as riskier.

Proof: We use the results proven in the former section. When women have the same salaries as men, both bring to the family the same income for the same hours in the marketplace. By (4.13) their productivity is the same at an optimum, and given the assumptions, it is more productive for both men and women to work the same hours in the marketplace. At the same time, by proposition 1 women work at home the same number of hours as men, since under the conditions, sharing work equally at home provides more family services for the same total amount of labour.

Reciprocally, when women and men share work equally at home, then from (4.13) it is optimal for the firm to pay both equally. The fair-fair pair of strategies just described is a Nash equilibrium of the market-family game because when following such a pair of strategies, each player is responding optimally to the others' move.

At a Nash equilibrium where women's salaries are inferior to men's, it is optimal for the family to choose an unfair distribution of household work by proposition 2. Women work more at home, and their productivity at home is lower as shown in proposition 2 and in the section entitled *Inequality lowers family welfare*, and so is their productivity at work by (4.13). This is an unfair-unfair Nash equilibrium, with both players responding optimally to each other. Nevertheless, it is a Pareto inferior solution.

The first fair-fair equilibrium is Pareto optimal. The following section illustrates why the fair-fair equilibrium is riskier under the conditions.

Illustration: a matrix game

The matrix below illustrates a game where the horizontal strategies represent the market's and the vertical represent the family's. The payoffs for the market are sub-indexed 1 and those for the family are sub-indexed 2.

$$\begin{pmatrix} & w_1 \neq w_2 & w_1 = w_2 \\ l_1 \neq l_2 & (A_1, A_2) & (C_1, D_2) \\ l_1 = l_2 & (D_1, C_2) & (B_1, B_2) \end{pmatrix}$$

In this matrix game, proposition 8 can be summarized by the inequalities

$$C_1 < A_1 \geq B_1 < D_1$$

and

$$C_2 < A_2 < B_2 < D_2$$

when (A_1, A_2) is the outcome of the unfair-unfair Nash equilibrium, (B_1, B_2) is the outcome of the fair-fair Nash equilibrium. The fair-fair Nash equilibrium is Pareto efficient because $A_1 < B_1$ and $A_2 < B_2$.

The Pareto efficient Nash equilibrium is more risky, because $C_1 < A_1$ so if the market plays fair but the family plays unfair the market will be worse off, this is proposition 3. Conversely, $C_2 < A_2$ implies that the family will be worse off if it plays fair while the market plays unfair, by proposition 2.

The family-market game is similar to prisoner's dilemma

The matrix presented above is similar to that of the 'prisoner's dilemma game' when in addition to the inequalities:

$$C_1 < A_1 < B_1 < D_1$$

and

$$C_2 < A_2 < B_2 < D_2$$

the two players are symmetrically situated, so that

$$A_1 = A_2, B_1 = B_2, C_1 = C_2, D_1 = D_2$$

A numerical example of the prisoner's dilemma is

$$\begin{pmatrix} 5, 5 & 3, 10 \\ 10, 3 & 9, 9 \end{pmatrix}$$

while a numerical example of our situation need not be symmetrical – for example

$$\begin{pmatrix} 5, 6 & 3, 10 \\ 9, 4 & 8, 9 \end{pmatrix}$$

where

$$\begin{aligned} A_1 &= 5 & A_2 &= 6 \\ B_1 &= 8 & B_2 &= 9 \\ C_1 &= 3 & C_2 &= 4 \\ D_1 &= 9 & D_2 &= 10 \end{aligned}$$

Conclusion

We show that the coupling of two distinct institutions – the market and the family – can lead to asymmetric allocations of effort and of rewards to two identical groups of people, men and women. In principle the asymmetries could be in favour of either group. However, given initial conditions, historically given differences in wages that favour men, the coupling leads to a rational but inferior solution that involves a disproportionate allocation of home responsibilities to women, and simultaneously to lower women's wages in the marketplace. However, as we showed, there is a cooperative solution that is better for all, involving equity at home and in the workplace. This latter solution is Pareto superior – but it seems riskier. The risks derive from missing contracts between the family and the marketplace. The family loses if it plays fair when the market doesn't, and vice versa (Edin and Richardson 2002; Elul *et al.* 2002; Engineer and Welling 1999).

What social institutions can help resolve this problem? Waldfogel (1998) and others have considered similar issues.

A prenuptial agreement that specifies women and men's roles in the family could be a start. It should have penalty, for example through 'bonds' that are posted in advance, if the parties default from what was promised initially. Using such a legal agreement women can present themselves at work as fully able to deliver so a fair employer is not misled about the nature of the labour it hires.

Similarly, strengthening equal pay provisions in the marketplace should support the execution of these prenuptial agreements. This requires enforcing the *Equal Pay Act* – and perhaps making this enforcement contingent on the availability of the prenuptial agreement just discussed. This way the firms would not risk being penalized for playing fair.

Other solutions to the prisoner's dilemma have been proposed over the years, most of them encourage cooperation among the players. Often this requires

repeated games among the players, which is not realistic in the case of marriage over small single digits (Lagerlöf 2003). In any case, any solution that encourages a cooperative outcome between the family and the market will benefit both. The moral of this article is that equity may appear to be riskier – and indeed, it may be – but it is after all the Pareto efficient allocation.

Another approach, suggested by O. Hart (in private communication), is to consider what informational structures within the firm are more likely to produce incentives that lead to equal pay to equal labour. It has been argued that piece-rates, when women are 'residual claimants', could do the job. An example would be women who own so-called 'franchises', where after paying a fixed rate for the name and the fixed costs, women act as self-employed and pay themselves the marginal product of labour. If this works, we should observe no gender gap in firms of this nature. This becomes an empirical question to be determined in future work.

In any case, the results of this article suggest that the gender gap is more problematic in firms where performance is difficult to observe, and that women who have more obligations at home – for example those who take care of children – face a larger gender gap. The existing empirical evidence for this seems reasonably good. Similarly, executive jobs that demand constant dedication would be most affected by the externality produced by the family that is mentioned in this article, and therefore in such jobs one would expect a glass ceiling to emerge: the higher a woman's level of accomplishment, the larger the gap between her salary and those of men. This glass ceiling is a direct implication of the results of this chapter and is extensively documented in the empirical literature. The empirical results discussed are validated by recent empirical work in Chichilnisky and Shachmurove (2007) based on data from the American Time Use Survey (ATUS).

Appendix

Proof of the existence of solution in proposition 7.

We show existence of a solution in a simple case; the most general case requires the use of a fixed point argument. The simplest (non trivial) case is when $\frac{w_1}{w_2} > M$ as defined in (M). Under the conditions, as we saw in proposition 2, women will do all the housework and men will only work in the marketplace. From (4.11) we obtain the total amount of hours that women work at home, denoted L_2 , which as already discussed, produces an externality on the productivity of women at the firm. There is no externality in the case of men, since men do not work at home. Therefore the total amount of hours that men work at the firm is L_1 and is determined from (4.11) and so is the marginal productivity $\frac{\partial f}{\partial L_1}$. Since we know the ratio of wages $\frac{w_1}{w_2}$ from (4.11) we may now derive the number of hours L_2 that women work at the firm together with the value of the externality γ – the two values L_2 and γ must satisfy the following two equations

$$\begin{aligned} \frac{w_1}{w_2} &= \frac{\frac{\partial f}{\partial L_1}(\gamma)}{\frac{\partial f}{\partial L_2}(\gamma)}. \end{aligned} \quad (4.20)$$

and

$$K - L_2 = l_2. \quad (4.21)$$

To solve the model we need to find the values of the two variables, γ^* and L_2^* , that satisfy the two equations (4.1) and (4.2). One shifts the production function using the externality parameter γ until the two equations are satisfied. At a solution, the productivity of women at the firm will be lower than men's, since women work most of their time at home. The vector $(w_1^*, w_2^*, \gamma^*, l_1^*, l_2^*)$ is a solution for this economy.

Notes

- 1 Graciela Chichilnisky is the UNESCO Professor of Mathematics and Economics at Columbia University and a Professor of Statistics at Columbia, and the Director of Columbia Consortium for Risk Management. This paper was presented at the economic theory Workshop of Columbia University in 2004, and at the June 2005 International School for Economic Research (ISER) on "Gender and Economics" at the University of Siena, <http://www.wcon-pol.unisi.it/iser.html>. It appeared first in the book "Gender and Economics" edited by Professor F. Bettio and G. Forconi (Chichilnisky 2005), and has been validated empirically in Chichilnisky and Shachmurove (2007). The author thanks the participants of the 2005 ISER for valuable comments and suggestions, and Oliver Hart for valuable insights on incentives and contracts in this model of gender discrimination.
- 2 The problem persists across all occupations and income levels, and is typically worse at the top. See Blay and Kahn (2004), Ginther (2004), Rosholm and Smith (1996), NCES (2004), Meyerson Milgrom *et al.* (2001), Meyerson Milgrom and Petersen (2003), Bureau of Labor Statistics (2003), Gupta *et al.* (2003).
- 3 In economic terms, there are externalities between the market and the family because the more persons work at home, the less reliable or productive they can be in the marketplace. In legal terms, there are missing property rights and missing contracts between the two institutions. Both of these issues impede the work of the market; they tie down the invisible hand.
- 4 Health service is an important sector, representing about 16% of the US GDP.
- 5 To clarify this issue their experiments (Gneezy *et al.* 2003) should be augmented to ask the women and the men who participate the amount of time they spend in each of the two institutions. In the case of students, the question may be better posed in terms of the amount of time they expect to spend on each of the two institutions - or the amount of time that their 'gender role models' - such as parents of teachers - themselves spend at home and in the marketplace.
- 6 Holmström and Milgrom (1991) examine people who share their time among different activities and predict specialization as does Becker. Their production functions have increasing productivity, and as a result each task is the responsibility of a single person thus predicting hierarchies. Under our conditions, instead, we show that at higher levels of employment equal sharing at home and at the marketplace emerges as the more productive strategy. This increases family welfare, and is more productive in the workplace.
- 7 In the following without loss we assume that the parameter is the same for the genders, $\gamma_1 = \gamma_2$.
- 8 Without loss of generality, we have assumed $\gamma_1 = \gamma_2$.
- 9 The consideration of leisure in the utility function would reinforce the results obtained in this chapter.
- 10 Recall that we normalized the price of x so that $p_x = 1$.

References

- Apps, P. F. and Rees, R. (1997) Collective Labor Supply and Household Production, *Journal of Political Economy*, 105 (1): 178-190.
- Aronsson, T., Daunfeldt, S.-O. and Wikström, M. (2001) Estimating Intrahousehold Allocation in a Collective Model with Household Production, *Journal of Population Economics*, 14(Part 4): 569-584.
- Arrow, Kenneth (1962) Economic Implications of Learning by Doing, *Review of Economic and Statistics* 29: 155-173.
- Becker, G. S. (1985) Human Capital, Effort, and the Sexual Division of Labor, *Journal of Labor Economics*, 3 (1): 33-58.
- Blau, F. D. and Kahn, L. M. (2004) *The US Gender Pay Gap in the 1990's: Slowing Convergence*, NBER Working Paper Series, Working Paper 10853.
- Bonke, J., Gupta, N. D. and Smith, N. (2005) Timing and Flexibility of Housework and Men and Women's Wages, IZA Discussion Paper no. 860, published in D. S. Hamermesh and G. A. Pfann (eds) *The Economics of Time use in Contributions to Economic Analysis*, Amsterdam: Elsevier, 2005.
- Bureau of Labor Statistics (2003) *Highlights of Women's Earnings in 2002*, Report 972, U.S. Department of Labor.
- Chichilnisky, G. (2005) The Gender Gap, Presented at the 2004-2005 Economic Theory Seminar of Columbia University and the CSWEP Session on Gender Discrimination at the 2005 American Economic Association Annual Meetings, Philadelphia, Pennsylvania, circulated in 2005 and 2006 as Working Paper of the Department of Economics, Columbia University, and published in F. Bettio and G. Forconi (eds.) *Gender and Economics, Proceedings of the 2005 ISER Conference*, University of Siena, June 2005. Available: <http://www.econ-pol.unisi.it/iser.html> (Accessed 27 February 2007).
- Chichilnisky, G. and Shachmurove, Y. (2007) *Household Work and the Gender Gap: an Empirical Analysis of Gender Discrimination*, Working Paper, Columbia University, Graduate Center CUNY and University of Pennsylvania, February 2007.
- Edin, P.-A. and Richardson, K. (2002) Swimming with the Tide: Solidary Wage Policy and the Gender Earnings Gap, *Scandinavian Journal of Economics*, 104 (1): 49-67.
- Elul, R., Silva-Reus, J. and Volij, O. (2002) Will you marry me? A Perspective on the Gender Gap, *Journal of Economic Behaviour and Organization*, 49: 549-572.
- Engineer, M. and Welling L. (1999) Human Capital, True Love, and Gender Roles: is sex destiny? *Journal of Economic Behaviour and Organization*, vol. 40: 155-178.
- Ginther, D. K. (2004) Why Women Earn Less: Economic Explanations for the Gender Salary Gap in Science, *Awis Magazine*, 33 (1): 6-10.
- Gneezy, U., Niederle, M. and Rustichini, M. (2003) Performance in Competitive Environments: Gender Differences, *The Quarterly Journal of Economics*, August 2003: 1049-1074.
- Gupta, N. D., Oaxaca, R.L. and Smith, N. (2003) *Swimming Upstream, Floating Downstream: Comparing Women's Relative Wage Positions in the US and Denmark*, International and labour relations review, 59(2): 243-266.
- Holmstrom, B. and Milgrom, P. (1991) Multitask Principal - Agent Analyses: Incentive Contracts, Asset Ownership and Job Design, *Journal of Law and Economic Organization*, 7 (Spring): 24-53.
- Lagerlöf, N.-P. (2003) Gender Equality and Long-Run Growth, *Journal of Economic Growth*, 8: 403-426.

- Meyersson Milgrom, E.-M., Petersen, T. and Snarland, V. (2001) Equal Pay for Equal Work? Evidence from Sweden and a Comparison with Norway and the US. *Scandinavian Journal of Economics* 103 (4): 559-583.
- Meyersson Milgrom, E.-M. and Petersen, T. (2006) Is There a Glass Ceiling for Women in Sweden, 1970-1990? Life Cycle and/or Cohort Effects. In *The Declining Significance of Gender?* edited by F. Blau, M. Brinton, and D. Grusky. New York: Russel Sage Foundation.
- NCES (2004) *Trends in Educational Equity of Girls & Women: 2004*. U.S. Department of Education, Institute of Education of Education Sciences, NCES 2005-016.
- Rosholm, M. and Smith, N. (1996) The Danish Gender Wage Gap in the 1980s: A Panel Data Study. *Oxford Economic Papers*, 48 (2): 254-279.
- Waldfogel, J. (1998) The Family Gap for Young Women in the United States and Britain: Can Maternity Leave Make a Difference? *Journal of Labor Economics*, 16 (3): 505-545.

5 Ghosts in the machine

A post Keynesian analysis of gender relations, households and macroeconomics

A. Haroon Akram-Lodhi and Lucia C. Hanmer

Introduction

Gender relations are a set of social norms, values, conventions and rules that informally or formally regulate the parameters of the practical day-to-day relationships between men and women within a society (Akram-Lodhi 1996). There is however one parameter of gender relations that apparently seems to transcend the myriad and multifaceted complex of gender-based cultural differences that can be found across societies and, as such, can claim the status of a 'stylized fact' of gender analysis. That parameter is that there are systemic asymmetries of social power between men and women, to the benefit of men. These systemic asymmetries are constructed on the basis of dominant gender ideologies that emphasize those aspects of life experience that differ between men and women. The power of gender ideology lies most fundamentally in its capacity to conflate the biological with the social and thus render as 'natural' the allocation of tasks by gender. Biological sex is a powerful, available metaphor for organizing society, generating a system of symbols which can interact with social institutions to asymmetrically structure relationships between men and women. Biological difference is thus used in the construction of a subjectivity that invests shared experience with different meanings, and in so doing becomes transformed into gender ideologies that shape cultural and social norms and, in turn, affect and effect material practices (Akram-Lodhi 1992a). The most notable material impact of gender ideologies is in the division of labour within the household, where women have a distinct role in performing the caring, maintenance and service activities that can be said to comprise 'household production'. At its most minimal, these activities can consist of the biologically-necessary tasks of food preparation, child care, sanitation and family reproduction.

The arena within which gender relations are played out, in the first instance, is thus the household. The household is

a social institution that embodies a particular pattern of relationships among individuals as biological and social beings...(and) an ideological concept through which people express their ideals about how biological and social