# What Is Sustainable Development?

***Graciela Chichilnisky***

ABSTRACT. *This paper introduces two axioms that capture the idea of sustainable development, and characterizes the welfare criterion that they imply. The axioms require that neither the present nor the future should play a dictatorial role in society's choices over time. Theorems 1 and 2 show there exist* sustainable preferences *which satisfy these axioms and provide a full characterization. Theorems 3–5 study a standard dynamical system representing the growth of a renewable-resource economy, give a "turnpike" theorem, and exhibit the differences between sustainable optima and the ones according to discounted utilitarianism.* (JEL O13)

## I. A VORACIOUS USE OF RESOURCES

For the first time in history, human activity has reached levels at which it could alter the planet's climate and its biological mix. Economics is the driving force. Energy used for production is obtained by burning fossil fuels, and leads to carbon emissions. The emissions generated since the second world war could alter the earth's climate, although there is as yet no scientific agreement on the precise magnitude of the effects. Biologists see the loss of biodiversity during the last fifty years as one of the four or five largest incidents of destruction of life on the planet. Originating largely in the industrial countries, this voracious use of resources has been accompanied by increasing discrepancies in resource consumption and welfare between industrial and developing countries. At the 1992 United Nations Earth Summit in Rio de Janeiro, *sustainable development* emerged as one of the most urgent subjects for international policy. One hundred and fifty participating nations endorsed UN Agenda 21, proposing as part of its policy agenda sustainable development based on the satisfaction of basic needs in developing countries. This development criterion was introduced by us in 1976 in the *Bariloche Model*[1] and given further impetus in 1987 when the Brundtland Commission proposed that "sustainable development is development that satisfies the needs of the present without compromising the needs of the future."[2] But *what is sustainable development*?

Solow (1992) pointed out recently that discussion of sustainability has been mainly an occasion for the expression of emotions and attitudes, with very little formal analysis of sustainability or of sustainable paths for a modern industrial economy. Formal analysis requires valuation, and the economic value of a resource is usually derived from its contribution to utility. This suggests that the crux of the matter is how to describe value so that it does not underestimate the future's interests and utilities, so that the future is given an equal treatment. This could be achieved in several ways. The challenge, however, is to develop economic theory which formalizes this aim with the level of clarity and substance achieved by neoclassical growth theory, and with the practical scope of the current approach to cost benefit analysis that is based on neoclassical growth theory.

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*I.A. Costs Today, Benefits Tomorrow*

A well-known problem is that standard cost-benefit analysis discounts the future. It is therefore biased against policies designed to provide benefits in the very long run. A sharp example is the evaluation of projects for the safe disposal of waste from a nuclear power plant. Another is policies designed for the prevention of global warming. The benefits of both may be at least fifty to a hundred years into the future. The costs, however, are here today. In these cases, the inherent asymmetry between the treatment of present and future makes it hard to justify investment decisions that large numbers of individuals and organizations clearly feel are well merited. Recent experimental evidence sheds new light on the matter.

*I.B. Experimental Evidence*

Several experiments have measured how people value the long run (see, e.g., Lowenstein and Thaler 1989, Cropper, Aydede, and Portney 1994, and the references in Lowenstein and Elster 1992). Their findings clash with the traditional discounted approach. People are shown to value the present and the future differently, but not as the standard analysis would predict. The experimental evidence indicates that the present and the future are treated more even-handedly. Typically we do discount the future, but the trade-off between today and tomorrow blurs as we move into the future. Tomorrow acquires increasing relative importance as time progresses. It is as if we viewed the future through a curved lens. The relative weight given to two subsequent periods in the future is inversely related to their distance from today. The period-to-period rate of discount is inversely related to the distance into the future. The experimental evidence shows that rate of discount between period $t$ and period $t + 1$ decreases with $t$. Interestingly, studies of human responses to sound summarized in the Weber-Fechner law, indicate similar responses to changes in sound intensity. The human ear responds to sound stimuli in an inverse relation to the initial stimulus.

How to explain this experimental evidence? How to make sense of our sensitivity to time and integrate it into a criterion of optimality? Several interesting alternatives to the discounted utility analysis have been proposed. So far none had reached the clarity and consistency of the discounted utilitarian criterion used in cost-benefit analysis, nor its analytical tractability. Prominent examples are the "overtaking criterion" and Ramsey's criterion. Both, however, are seriously incomplete, failing to rank many reasonable paths. The ordering induced by the overtaking criterion cannot be represented by a real valued function, making it impractical to use. As a result, they lack the corresponding "shadow" prices to evaluate costs and benefits in an impartial fashion. These criteria therefore fail on practical grounds.

*I.C. A Criterion for Sustainability*

This paper proposes simple axioms which capture the concept of sustainability, and derives the welfare criterion which they imply, see also Chichilnisky (1996). The criterion that emerges is complete, analytically tractable, and represented by a real valued function. In optimization it leads to well-defined shadow prices which can be used for a "sustainable cost-benefit analysis." The axioms provide internal consistency and ethical clarity. They imply a more symmetric treatment of generations in the sense that neither the "present" nor the "future" should be favored over the other. They neither accept the romantic view which relishes the future without regards for the present, nor the consumerist view which ranks the present above all. The axioms lead to a complete characterization of *sustainable preferences*,[3] which are sensitive to the welfare of all generations, and offer an equal opportunity to the present and to the future. Trade-offs between present and future consumption are allowed. The existence and characterization of sustainable preferences appears in Theorems 1 and 2. Theorem 1

[3] An alternative name suggested by Robert Solow is "intertemporally equitable preferences."

shows that sustainable preferences are different from all other criteria used so far in the analysis of optimal growth and of markets. Theorem 3 studies a standard dynamical system representing the growth of a renewable resource. Sustainable preferences are shown to be a natural extension of the "equal treatment criterion" for finitely many generations, in the sense that the optimal solutions for such preferences approach the "turnpike" of an equal-weight finite horizon optimization problem as the horizon increases. Theorem 3 also shows that sustainable preferences match the experimental evidence in these cases, in the sense that they imply a rate of discount that decreases and approaches zero as time goes to infinity. Theorem 4 investigates the relationship between the optimal paths according to sustainable preferences and discounted utilitarianism in an extension of the classical Hotelling problem of the optimal depletion of an exhaustible resource. Theorem 5 shows that sustainable optima can be quite different from discounted optima, no matter how small is the discount factor. Subsequent examples show the implications for shadow prices.

## II. TWO AXIOMS FOR SUSTAINABLE DEVELOPMENT

The two following axioms are non-dictatorship properties.[4] Axiom 1 requires that the present should not dictate the outcome in disregard for the future: it requires sensitivity to the welfare of generations in the distant future. Axiom 2 requires that the welfare criterion should not be dictated by the long-run future, and thus requires sensitivity to the present.[5] To offer a formal perspective a few definitions are required.

Each generation is represented by an integer $g$, $g = 1, 2, \ldots, \infty$. An infinitely lived world obviates the need to make decisions contingent on an unknown terminal date. Generations could overlap or not. Indeed agents could be infinitely long-lived and evaluate development paths for their own futures.

For ease of comparison, I adopt a formulation which is as close as possible to that of the neoclassical model. Each generation $g$ has a preference that can be represented by a utility function $u_g$ for consumption of $n$ goods, some of which could be environmental goods such as water, or soil, so that consumption vectors are in $R^n$, and $u_g$: $R^n \to R$. The availability of goods in the economy is constrained in a number of ways, for example by a differential equation which represents the growth of the stock of a renewable resource,[6] and/or the accumulation and depreciation of capital. Ignore for the moment population growth, although this issue can be incorporated at the cost of simplicity, but with little change in the results.[7] The space of all feasible consumption paths is indicated by $F$.

$$F \subset \{x: x = \{x_g\}_{g=1,2,\ldots}, x_g \in R^n\}. \quad [1]$$

We chose a utility representation so that each generation's utility function is bounded below and above: $u_g$: $R^n \to R^+$, and $\sup_{x \in R^N}(u_g(x)) < \infty$. This choice is not restrictive: it was shown by Arrow (1964) that when ranking infinite streams of utilities as done here one should work with bounded utility representations since doing otherwise could lead to paradoxes.[8] Utility across generations is assumed to be comparable. In order to eliminate some of the most obvious problems of comparability I normalize the

[4] See Arrow (1953), Chichilnisky (1994). In this case we are concerned with fairness across generations, see also Solow (1992), Lauwers (1993, 1997).

[5] Chichilnisky (1982, 1994, 1996).

[6] See Chichilnisky (1993c) and Beltratti, Chichilnisky, and Heal (1995).

[7] Population growth and utilitarian analysis are well known to make an explosive mix, which is however outside the scope of this paper.

[8] A preference admits more than one utility representation; among these one chooses a bounded representation. The need to work with bounded utility representation in models with infinitely many parameters was pointed out by Arrow (1964), who required boundedness to solve the problem that originally gave rise to Daniel Bernouilli's famous paper on the "St. Petersburg paradox," *Utility Boundedness Theorem* (Arrow 1964, 27). If utilities are not bounded, one can find a utility stream for all generations with as large a welfare value as we wish, and this violates standard continuity axioms.

utility functions $u_g$ so that they are non-negative and all share a common bound, which I assume without loss of generality to be 1:

$$\sup_g (u_g(x_g))_{x_g \in R^n} \leq 1. \quad [2]$$

The space of *feasible utility streams* $\Omega$ is therefore

$$\Omega = \{\alpha : \alpha = \{\alpha_g\}_{g=1,2,\ldots},\ \alpha_g = u_g(x_g)\}_{g=1,2,\ldots} \text{ and } x = \{x_g\}_{g=1,2,\ldots} \in F\}. \quad [3]$$

Each utility stream is a sequence of positive real numbers bounded by the number 1. The space of all utility streams is therefore contained in the space of all infinite bounded sequences of real numbers, denoted $\ell_\infty$.[9] Our welfare criterion $W$ should rank elements of $\Omega$, for all possible $\Omega \subset \ell_\infty$.

*II.A. Sensitivity and Completeness*

The welfare criterion $W$ is *complete* if it is represented by an increasing real valued function defined on all bounded utility streams[10] $W$: $\ell_\infty \to R^+$. It is *sensitive*, or increasing, if whenever a utility stream $\alpha$ is obtained from another $\beta$ by increasing the welfare of some generation, then $W$ ranks $\alpha$ strictly higher than $\beta$.[11]

*II.B. The Present*

How to represent the present? Intuitively, the present is represented by all the utility streams which have no future: for any given utility stream $\alpha$, its "present" is represented by all finite utility steams which are obtained by cutting $\alpha$ off after any number of generations. Formally:

DEFINITION 1. *For any utility stream $\alpha \in \ell_\infty$, and any integer K, let $\alpha^K$ be the "K-cutoff" of the sequence $\alpha$, the sequence whose coordinates up to and including the K-th are equal to those of $\alpha$, and zero after the K-th.*[12]

DEFINITION 2. *The present consists of all feasible utility streams which have no future, i.e., it consists of the cutoffs of all utility streams.*

*II.C. The Future*

By analogy, for any given utility stream $\alpha$, its "future" is represented by all infinite utility streams which are obtained as the "tail" resulting from cutting $\alpha$ off for any finite number of generations.

DEFINITION 3. *The K-th tail of $\gamma$ is the sequence whose coordinates up to and including the K-th are zero and equal to those of $\gamma$ after the K-th generation.*[13]

For any two $\alpha, \gamma \in \ell_\infty$, let $(\alpha^K, \gamma_K)$ be the sequence defined by summing up or "pasting together" the $K$-th cutoff of $\alpha$ with the $K$-th tail of $\gamma$.

*II.D. No Dictatorial Role for the Present*

DEFINITION 4. *A welfare function $W$: $\ell_\infty \to R$ gives a dictatorial role to the present, and is called a dictatorship of the present, if W is insensitive to the utility levels of all but a finite number of generations, i.e., if W is only sensitive to the "cutoffs" of utility streams, and it disregards the utility levels of all generations from some generation on.*

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[9] Formally: $\Omega \subset \ell_\infty$ where $\ell_\infty = \{y: y = \{y_g\}_{g=1,\ldots}: y_g \in R^+, \sup_g |y_g| < \infty\}$. Here $|\cdot|$ denotes the absolute value of $y \in R$, which is used to endow $\ell_\infty$ with a standard Banach space structure, defined by the norm $\|\cdot\|$ in $\ell_\infty$

$$\|y\| = \sup_{g=1,2,\ldots} |y_g|. \quad [4]$$

The space of sequences $\ell_\infty$ was first used in economics by Debreu (1954).

[10] The representability of the order $W$ by a real valued function can be obtained from more primitive assumptions, such as, for example, transitivity, completeness, and continuity conditions on $W$.

[11] Formally: if $\alpha > \beta$ when $W(\alpha) > W(\beta)$.

[12] In symbols: $\alpha^K = \{\sigma_g\}_{g=1,2,\ldots}$ such that $\sigma_g = \alpha_g$ if $g \leq K$, and $\sigma_g = 0$ if $g > K$.

[13] In symbols: $\alpha_K = \{\sigma_g\}_{g=1,2,\ldots}$ such that $\sigma_g = 0$ if $g \leq K$, and $\sigma_g = \alpha_g$ if $g > K$.

Formally, $W$ is a dictatorship of the present if for any two utility streams[14] $\alpha, \beta$

$$W(\alpha) > W(\beta) \Leftrightarrow$$

$$\exists N = N(\alpha, \beta)$$

$$\text{s.t. if } K > N, W(\alpha^K, \gamma_K) > W(\beta^K, \sigma_K)$$

$$\text{for any utility streams } \gamma, \sigma \in \ell_\infty.$$

The following axiom eliminates dictatorships of the present:

- **Axiom 1: No dictatorship of the present.**

*II.E. No Dictatorial Role for the Future*

DEFINITION 5. *A welfare function $W: \ell_\infty \to R$ gives a dictatorial role to the future, and is called a dictatorship of the future, if $W$ is insensitive to the utility levels of any finite number of generations, or equivalently it is only sensitive to the utility levels of the "tails" of utility streams.*

*Formally: for every two utility streams $\alpha, \beta$*

$$W(\alpha) > W(\beta) \Leftrightarrow$$

$$\exists N = N(\alpha, \beta.): \text{ if } K > N,^{15}$$

$$W(\gamma^K, \alpha_K) > W(\sigma^K, \beta_K), \quad \forall \gamma, \sigma \in \ell_\infty.$$

The welfare criterion $W$ is therefore only sensitive to the utilities of "tails" of streams, and in this sense the future always dictates the outcome independently of the present. The following axiom eliminates dictatorships of the future:

- **Axiom 2: No dictatorship of the future.**

DEFINITION 6. *A sustainable preference is a complete sensitive preference satisfying Axioms 1 and 2.*

## III. EXISTENCE AND CHARACTERIZATION OF SUSTAINABLE PREFERENCES

Why is it difficult to rank infinite utility streams? Ideally one would give equal weight to every generation. For example, with a finite number $N$ of generations, each generation can be assigned weight $1/N$. But when trying to extend this criterion to infinitely many generations one encounters the problem that, in the limit, every generation is given zero weight.

What is done usually to solve this problem is to attach more weight to the utility of near generations, and less weight to future ones. An example is of course the sum of discounted utilities. Discounted utilities give a bounded welfare level to every utility stream which assigns each generation the same utility. Two numbers can always be compared, so that the criterion so defined is clearly complete. However, the sum of discounted utilities is not even-handed: it disregards the long-run future. I show below that it is a dictatorship of the present.

Another solution is the criterion defined by the long-run average of a utility stream, a criterion used frequently in repeated games. However, this criterion is not even-handed either: it is biased in favor of the future and against the present. It is insensitive to the welfare of any finite number of generations.[16]

Here matters stood for some time. Asking for the two axioms together, the non-dictatorship of the present and the non-dictatorship of the future, as I do there, appears almost as if it would lead to an impossibility theorem. Not quite.

Let us reason again by analogy with the case of finite generations. To any finite

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14. Recall that all utility streams are in $\ell_\infty$ and they are normalized so that $\sup_{g=1,2,\ldots}(\alpha(g)) = \|\alpha\| \leq 1$ and $\sup_{g=1,2,\ldots}(\beta(g)) = \|\beta\| \leq 1$.

15. An equivalent definition for our purposes would be obtained by replacing "$\exists N$" by "for any $K$."

16. Other interesting incomplete intergenerational criteria which have otherwise points in common with sustainable preferences are found in Asheim (1988, 1991)

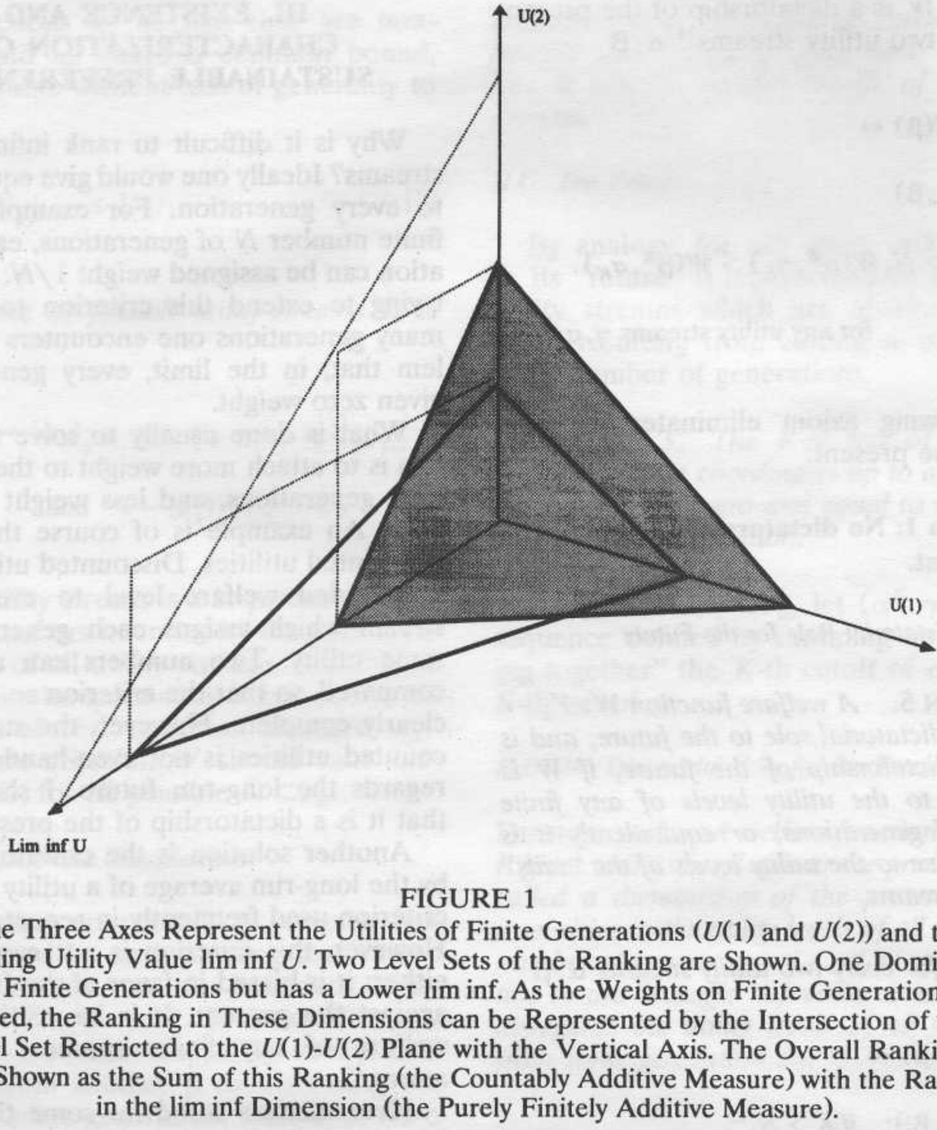


FIGURE 1
The Three Axes Represent the Utilities of Finite Generations ($U(1)$ and $U(2)$) and the Limiting Utility Value lim inf $U$. Two Level Sets of the Ranking are Shown. One Dominates Over Finite Generations but has a Lower lim inf. As the Weights on Finite Generations are Fixed, the Ranking in These Dimensions can be Represented by the Intersection of the Level Set Restricted to the $U(1)$-$U(2)$ Plane with the Vertical Axis. The Overall Ranking is then Shown as the Sum of this Ranking (the Countably Additive Measure) with the Ranking in the lim inf Dimension (the Purely Finitely Additive Measure).

number of generations one can assign weights which decline into the future, and then assign some extra weight to the last generation. This procedure, when extended naturally to infinitely many generations, is neither dictatorial for the present nor for the future. It is similar to adding to a sum of discounted utilities, the long-run average of the whole utility stream. Neither part of the sum is acceptable on its own, but together they are. This is Theorem 1 below. Theorem 2 proves that under regularity conditions this procedure gives a complete characterization of all continuous sustainable preferences.

The first part of Theorem 1 establishes that the sum of a dictatorship of the present plus a dictatorship of the future is in fact neither. The first part is sensitive to the present, and the second is sensitive to the future. Furthermore such a sum admits trade-offs between the welfare of the present and of the future. It is represented diagrammatically in Figure 1, which shows the trade-offs between the present's and the future's utilities. The three axes represent the utility levels of generations 1, 2, and, figuratively, $\infty$. The two triangular planes represent two indifference surfaces. One gives more utility to generations 1 and 2, and under a dictatorship of the present these choices would prevail; however the second surface gives more utility to the long run, so that under certain conditions the second surface is chosen over the first. Theorem 1 makes this reasoning rigorous.

The second part of Theorem 1 shows that all known criteria of optimality used until now fail to satisfy the axioms postulated here. Therefore the sustainable preferences defined here perform a role satisfied by no previously used criterion.

What is perhaps more surprising is that the sustainable welfare criteria constructed here, namely the sum of a dictatorship of the present and one of the future, exhaust all the continuous utilities which satisfy my two axioms. This means that any continuous sustainable preference must be of the form just indicated. This is Theorem 2, proved in the Appendix.

### *III.A. The Existence of Sustainable Preferences*

THEOREM 1. *There exists a sustainable preference $W: \ell_\infty \to R$, i.e., a preference which is sensitive and does not assign a dictatorial role to either the present or the future*:

$$W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g + \phi(\alpha), \qquad [5]$$

*where $\forall g$, $\lambda_g > 0$, $\sum_{g=1}^{\infty} \lambda_g < \infty$, and where $\phi(\alpha)$ is the function $\lim_{g \to \infty}(\alpha_g)$ extended to all of $\ell_\infty$ via Hahn-Banach theorem.*[17]

*The following welfare criteria are not sustainable preferences*: (*a*) *the sum of discounted utilities, for any discount factor*, (*b*) *Ramsey's criterion*, (*c*) *the overtaking criterion*, (*d*) *lim inf*, (*e*) *long-run averages*, (*f*) *Rawlsian rules, and* (*g*) *basic needs*.

PROOF. In the Appendix.

An intuitive explanation of this result follows. The preference defined in [5] is sustainable because it is complete, its first term is sensitive to the present, in fact it increases with increases in the welfare of every generation, and its second term is sensitive to the long-run future. The next step is to show why all other criteria fail the sustainability axioms proposed here.

(a) The sum of discounted utilities is a dictatorship of the present because for every $\varepsilon > 0$, there exists a generation $N$ so that the sum of discounted utilities of all other generations beyond $N$ is lower than $\varepsilon$ for all utility streams since all utilities are bounded by the number 1, [2]. Now, given any two utility streams $\alpha, \beta$, if $W(\alpha) > W(\beta)$ then $W(\alpha) > W(\beta) + \varepsilon$ for some $\varepsilon > 0$; therefore there exists a generation $N$ beyond which the utilities achieved by any generation beyond $N$ do not count in the criterion $W$. This is true for any given discount factor.[18]

(b) The Appendix establishes that the Ramsey's criterion is incomplete; this derives from the fact that the distance to Ramsey's bliss path is ill-defined for many paths.

(c) The Appendix establishes that the overtaking criterion is incomplete: see also Figure 2.

(d) and (e) Lim inf and long-run averages are dictatorships of the future; furthermore the long-run average is also incomplete.[19]

Both (f) and (g), Rawlsian and basic needs criteria, are insensitive because they rank equally any two paths which have the same infimum even if one assigns much higher utility to many other generations.

### *III.B. A Complete Classification of Sustainable Preferences*

The following result characterizes sustainable preferences. Additional conditions

[17] The linear map "$\lim_{g \to \infty}(\alpha_g)$" is defined by using the Hahn-Banach theorem, as follows: define first the function on the closed subset of $\ell_\infty$ consisting of those sequences $\alpha_g$ which have a limit, as that limit; the function is then extended continuously to all sequences in the space $\ell_\infty$ by using the Hahn-Banach theorem, which ensures that such an extension exists and can be constructed while preserving the norm of the function on the closed subspace of convergent subsequences.

[18] Interestingly Solow has suggested making the discounting factor smaller than one and decreasing: it is possible to show that *under certain conditions* this is exactly what must be done to ensure the existence of solutions.

[19] For example the two sequences (1, 0, 0, 111, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, ...) and (0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, ...) are not comparable according to the long-run averages criterion.

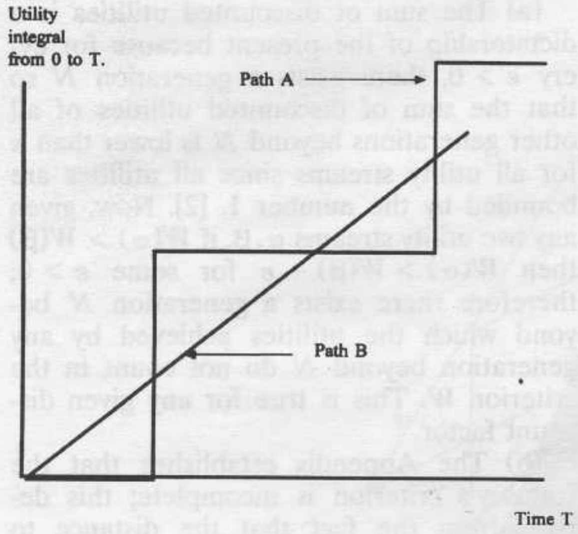


FIGURE 2
Neither Path Overtakes the Other, Illustrating the Incompleteness of the Overtaking Criterion.

on the welfare criterion $W$ are now introduced: $W$ is *continuous* when it is defined by a continuous function $W: \ell_\infty \rightarrow R$.[20] Continuity has played a useful role in social choice theory in the last ten years, in effect replacing the axiom of independence of irrelevant alternatives and allowing a complete characterization of domains in which social choice exists (Chichilnisky 1982, 1993a; Chichilnisky and Heal 1983). A similar role is found here for continuity: the following theorem gives a full characterization to all sustainable criteria which are continuous.

A standard property of neoclassical analysis is that the rate of substitution between two generations—which is generally dependent on their level of consumption—is independent of their levels of utility. This is a widely used property: indeed the sum of discounted utilities, the most widely used welfare criterion, certainly satisfies it. A welfare criterion $W$ satisfying this property is called *independent*.[21] The characterization of a sustainable criterion $W$ in the following theorem is simplest when $W$ is continuous and independent. In consistency with neoclassical analysis, we therefore assume independent and welfare criteria. The following theorem decomposes a sustainable criterion into the sum of two functions. The first is a discounted utility with a variable discount rate and the second is a generalization of long-run averages or a lim inf, and is called a "purely finitely additive" measure. For definitions and examples of purely finitely additive measures the reader is referred to the Appendix, Example 4: this latter measure assigns all welfare weight to the very long run. In particular, it assigns the value zero to any sequence which has finitely many non-zero terms.[22]

THEOREM 2. *Let $W: \ell_\infty \rightarrow R^+$ be a continuous independent sustainable preference. Then $W$ is of the form $\forall \alpha \in \ell_\infty$:*

$$W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g + \phi(\alpha) \qquad [6]$$

*where $\forall g\ \lambda_g > 0, \sum_{g=1}^{\infty} \lambda_g < \infty$, and $\phi \neq 0$ is a purely finitely additive measure.*

PROOF. In the Appendix.

## IV. ARE SUSTAINABLE PREFERENCES REASONABLE?

We saw that sustainable preferences emerge from well-defined and uncontrover-

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[20] A function $W$ which is continuous with respect to the standard norm of the space of sequences $\ell_\infty$. The norm is $\|\alpha\| = \sup_{g=1,2,\ldots} |\alpha_g|$, and was defined above. Different forms give rise to different notions of continuity but in the context of equitable treatment of generations the sup norm is a natural candidate.

[21] See the Appendix. This simply means that the indifference surfaces of the welfare criterion $W$ are hypersurfaces so that it is possible to represent it by a linear function on utility streams: $W(\alpha + \beta) = W(\alpha) + W(\beta)$. Note that this does not restrict the utilities of the generation, $u_g$, in any way; in particular the $u_g'$s need not be linear and so the marginal rate of substitution between consumption at different dates can be non-constant and depend *inter alia* on consumption levels.

[22] A finitely additive measure on the integers $Z$ is a function $\mu$ defined on subsets of the integers, satisfying $\mu(A \cup B) = \mu(A) + \mu(B)$ when $A \cap B = \varnothing$; $\mu$ if called purely finitely additive when it assigns measure zero to any finite subset of integers. See also the Appendix for definitions and examples.

sial axioms. But how do they fit economic intuition and empirical evidence? The following subsection will show that sustainable preferences fit well our economic intuition about finite horizon optimization. Subsection IV.B will show that they also fit the empirical evidence rather well.

*IV.A. A Turnpike Theorem*

Our economic intuition is grounded on finite horizons. Life on earth will certainly be of finite duration, although it is difficult to determine its final date. It is therefore important to determine whether sustainable preferences are merely an artifact of infinite horizons, or are reasonable within a finite world.

This section will show that sustainable preferences can be seen as a suitable generalization to infinite horizons of an intuitively appealing criterion for finite horizons, one which values all generations equally, thus providing "equal treatment." Indeed, for a general class of dynamic optimization problems, we will see that the limit of the optimal solution according to a sustainable preference has two interesting properties: (i) it is the "green golden rule" (see Beltratti, Chichilnisky, and Heal 1995), that is, the configuration of the economy giving the maximum sustainable utility level and (ii) as the finite horizon increases, the optimal solutions of equal treatment finite horizon problems spend an increasing amount of time progressively closer to the limit of the path which is optimal according to sustainable preferences. In other words, the optimum according to sustainable preferences determines a direction in which finite horizon equal treatment optima increasingly move as the horizon increases. We refer to this property as a "turnpike" property.

To see this we formalize a typical problem of optimizing a sustainable preference over a constraint imposed by the dynamics of a renewable stock. A renewable stock $s_t$ grows over time $t$ according to its own biological dynamics, with growth rate $r(s_t)$; $c_t$ of it is extracted for consumption. The utility depends on consumption and the level of the stock, as in for example Krautkraemer (1985) or Heal (1995). We are assuming the stock of the resource to be an argument of the utility function, so that the resource is in the category of environmental assets such as forests, landscapes, biodiversity, etc., which provide services and value to human society via their stocks as well as via a flow of consumption. The problem is therefore:

$$\left.\begin{aligned}&\max \alpha \int_0^\infty u(c_t, s_t) e^{-\delta(t)t}\, dt + (1-\alpha) \lim_{t\to\infty} u(c_t, s_t)\\ &\text{s.t. } \dot{s}_t = r(s_t) - c_t,\ s_0 \text{ given.}\end{aligned}\right\} \tag{7}$$

To study the asymptotic properties of a maximum for this problem it is useful to introduce the following definition, see also Beltratti, Chichilnisky, and Heal (1995):

DEFINITION 7. *The green golden rule $g^*$ is a stationary path $g^* = \{c^*, s^*\}$ which achieves the maximum utility level which is sustainable forever, that is,*

$$g^* = \max_c u(c, s)$$

*subject to $c \le R(s)$.*

*Equivalently*:

$$g^* = \max_s u(R(s), s)$$

*so that it satisfies*:

$$\frac{\partial u}{\partial c}\frac{\partial R}{\partial s} = -\frac{\partial u}{\partial s}, \quad \text{or} \quad \frac{\partial R}{\partial s} = -\frac{\frac{\partial u}{\partial s}}{\frac{\partial u}{\partial c}}. \tag{8}$$

DEFINITION 8. *The equal treatment problem for horizon T is*:

$$\left.\begin{aligned}&\max \int_0^T u(c_t, s_t)\, dt\\ &s.t.\ \dot{s}_t = r(s_t) - c_t,\ s_0 \text{ given.}\end{aligned}\right\} \tag{9}$$

Its solution is called the equal treatment optimum over $T$ generations.

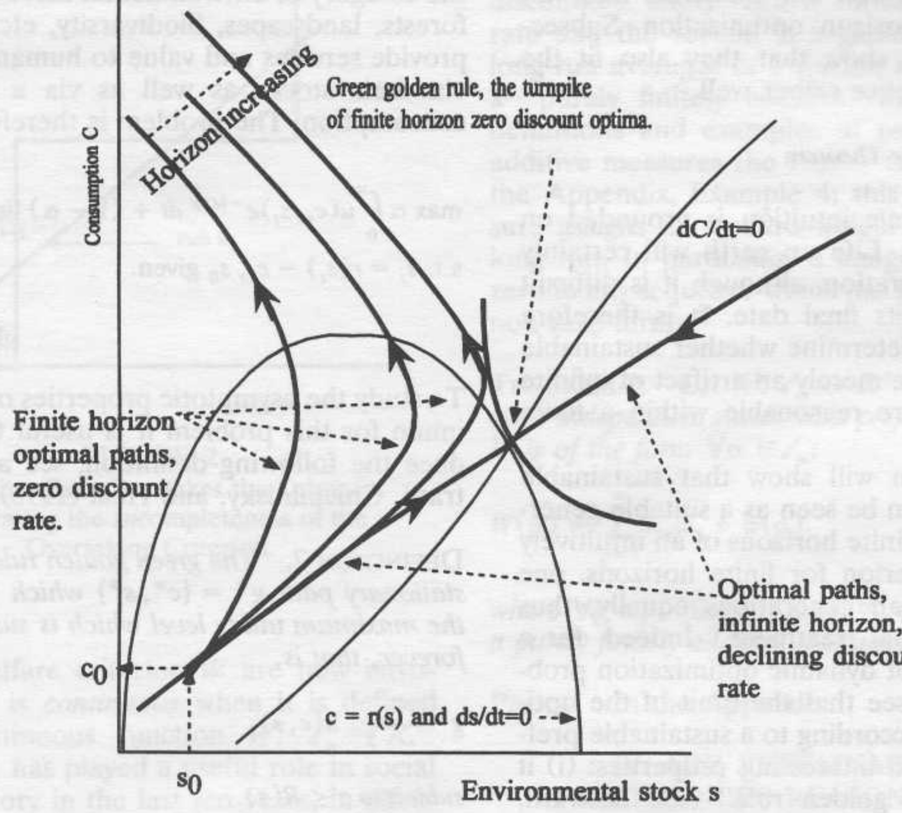


FIGURE 3
The Utilitarian Solution with a Renewable Resource, Stock an Argument of the Utility Function and Discount Rate Falling to Zero, Asymptotes to the Green Golden Rule. This is also the Turnpike of Finite Horizon Equal Treatment (Zero Discount) Optima.

DEFINITION 9. *The discounted utilitarian problem has the same constraint set as problem* [7] *but the function to be maximized is, instead, the integral of utilities discounted by a constant positive discount rate*:

$$\int_0^\infty u(c, s)e^{\delta t}\, dt, \quad \text{for a fixed } \delta > 0. \qquad [10]$$

THEOREM 3. *The optimal path for problem* [7] *with a sustainable preference exists if and only if the discount rate* $\delta(t)$ *approaches zero in the limit, in which case the optimum is the "turnpike" of finite horizon problem* [9] *in which each generation is treated equally. This means that as the number of generations T increases, the equal treatment optima for T generations are increasingly closer to the green golden rule, a plan which is asymptotically approached by the optima of* [7] *with sustainable preferences. Formally*:

(1) *The green golden rule* $g^*$ *has a "turnpike property" for equal treatment optima. That is, as* $T \to \infty$ *the distance between the equal treatment optimum for T and* $g^*_g\, \varepsilon(T)_g$ *goes to zero.*

(2) *The optimal solution to problem* [7] *exists if and only if* $\delta(t) \to 0$ *as* $t \to \infty$, *in which case it is a path* $(\hat{c}_t, \hat{s}_t)$ *which converges asymptotically to the green golden rule* $g^*$, *and so to the turnpike of the equal treatment optima.*

(3) *By contrast, the discounted utilitarian optimum does not have the turnpike property for equal treatment optima, for any positive (fixed) discount rate* $\delta$: *the optimal path for a positive discount rate* $\delta$ *is uniformly bounded away from* $g^*$ *and so from the turnpike of the equal treatment optima. For a zero discount rate, the discounted utility problem has no solution.*

PROOF. See Heal (1995).

Figure 3 illustrates the results. The green golden rule is the pair of consumption and stock levels at which the marginal conditions for static optimality [8] are satisfied. The solutions for the equal treatment criterion for finite time horizons $T$ are indicated: these solutions are found by noting that the stock at $T$ should be zero, and then finding the appropriate initial conditions by solving the Euler-Lagrange equations. As $T \to \infty$, these paths come closer and closer to the green golden rule. This is the turnpike property.

*IV.B. Empirical Evidence Matches Sustainable Preferences*

Sustainable preferences help explain the empirical evidence on time preferences. Recent empirical evidence on time preferences clashes with standard discounted utility maximization. However this evidence is consistent with the solutions obtained from optimizing sustainable preferences in the context of a renewable resource, in the sense that an optimum exists if and only if the discount rate falls over time, as in part (2) of Theorem 3.

There is a growing body of empirical evidence that suggests that the discount rate which people apply to future projects depends upon, and declines with, the futurity of the project. See for example, Lowenstein and Thaler (1989) or Cropper, Aydede, and Portney (1994). Over relatively short periods up to perhaps five years, they use discount rates which are higher even than commercial rates—in the region of 15 percent or more. For projects extending about ten years, the implied discount rates are closer to standard rates—perhaps 10 percent. As the horizon extends the implied discount rates drop, to in the region of 5 percent for thirty to fifty years and down to of the order of 2 percent for one hundred years. The evidence for these statements is still tentative, and more research is needed to document fully how people trade off the future against the present. However, our framework for intertemporal optimization has an implication for discounting that rationalizes a behavior that hitherto has been found irrational.

*IV.C. Weber-Fechner Law*

Well-known results from natural sciences establish that human responses to a change in a stimulus are nonlinear, and are inversely proportional to the existing level of the stimulus. For example, the human response to a change in the intensity of a sound is inversely proportional to the initial sound level: the louder the sound initially, the less we respond to a given increase. This is an example of the Weber-Fechner law, which can be formalized in the statement that human response to a change in a stimulus is inversely proportional to the preexisting stimulus. In symbols.

$$\frac{dr}{ds} = \frac{K}{s} \quad \text{or} \quad r = K \log s$$

where $r$ is a response, $s$ a stimulus, and $K$ is constant. This law has been found to apply to human responses to the intensity of both light and sound signals. The empirical results on discounting cited above suggest that something similar is happening in human responses to changes in the futurity of an event: a given change in futurity (e.g., postponement by one year) leads to a smaller response in terms of the decrease in weighting, the further the event already is in the future. In this case, the Weber-Fechner law can be applied to responses to distance in time, as well as to sound and light intensity, with the result that the discount rate is inversely proportional to distance into the future.

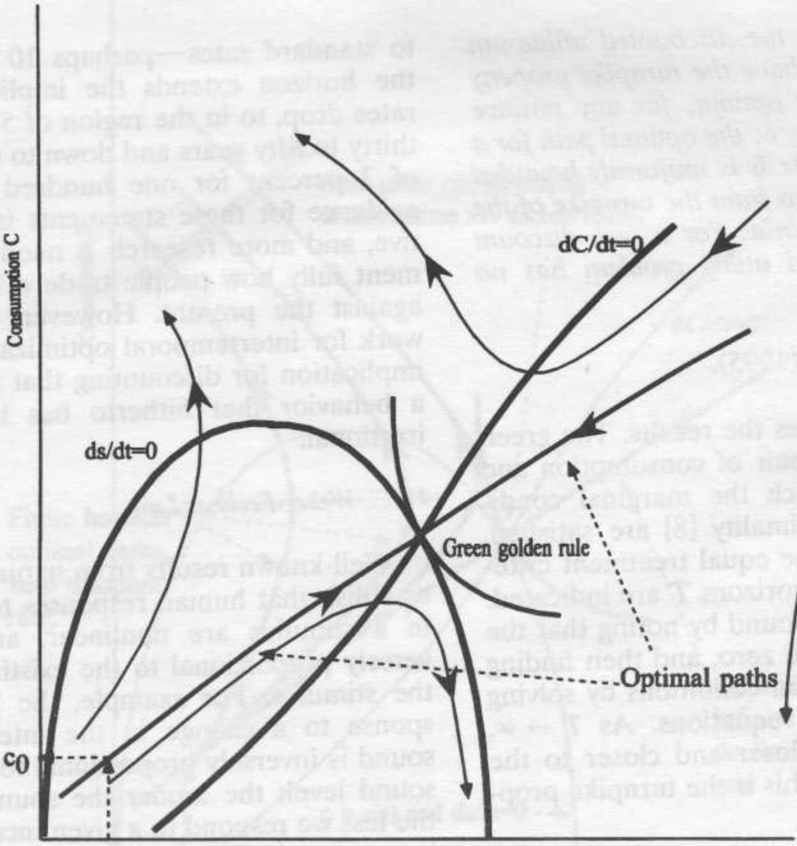


FIGURE 4
The Green Golden Rule is the Stationary Solution of the Autonomous System [7], and is a Saddle Point.

In our economic problem

$$\max \int_0^\infty u(c, s)\Delta(t)\, dt + \lim u(c, s),$$

the discount rate is $q(t) = \dot{\Delta}(t)/\Delta(t)$, where $\Delta(t)$ is the discount factor. One can formalize the Weber-Fechner interpretation as follows:

$$q(t) = \frac{1}{\Delta}\frac{d\Delta}{dt} = \frac{K}{t} \quad \text{or} \quad \Delta(t) = e^{K \log t} = t^K$$

where $K$ is a negative constant.

Such a discount factor meets all of the conditions required for the existence of sustainable optima: the discount rate $q$ goes to zero in the limit, the discount factor $\Delta(t)$ goes to zero, and the integral $\int_1^\infty \Delta(t)\, dt = \int_1^\infty e^{K \log t}\, dt = \int_1^\infty t^K\, dt$ converges for $K$ negative, as it always is.

A discount factor $\Delta(t) = e^{K \log t}$ has an interesting interpretation: the replacement of $t$ by $\log t$ implies that we are measuring time differently: by equal proportional increments rather than by equal absolute increments. This is consistent with the approach taken in, for example, acoustics, where in response to the Weber-Fechner law sound intensity is measured in decibals which respond to the logarithm of the energy content of the sound waves. In general, non-constant discount rates can be interpreted as a nonlinear transformation of the time axis.

## V. DIFFERENCES BETWEEN STANDARD AND SUSTAINABLE OPTIMA

### *V.A. Renewable Resources*

The next step in exploring the properties of sustainable preferences is to study the difference between sustainable and discounted utilitarian optima. We do this first for the case of optimization with renewable resources, comparing the optimal path for sustainable preferences, which solves [7] above, with that which is optimal for the integral of discounted utilities, namely [10] above. In fact most of the elements needed for this comparison are in place. Figure 4 shows the optimal path for sustainable preferences, which asymptotes to the green golden rule, namely the point of tangency between an indifference curve of the utility function $u(c, s)$ and the growth function $r(s)$. A stationary solution to the utilitarian case is characterized by the following conditions:

$$\left.\begin{aligned} r(s_t) &= c_t \\ \frac{u_2'(s_t)}{u_1'(c_t)} &= \delta - r'(s_t) \end{aligned}\right\}. \qquad [11]$$

The first equation in [11] just tells us that a stationary solution must lie on the curve on which consumption of the resource equals its renewal rate: this is obviously a prerequisite for a stationary stock. The second is derived from the standard conditions for dynamic optimization and it implies that the indifference curve cuts the renewal function from above as shown in Figure 5. This involves a lower long-run stock and a higher long-run level of consumption than the sustainable optimum, shown in Figure 4 and in this sense is less conservative: it also involves a lower long-run utility level. In the utilitarian solution for a constant discount rate, as the discount rate falls to zero, the stationary solution moves to the green golden rule defined in [8]. However, for a constant discount rate of zero, the utilitarian problem has no solution. Note that in

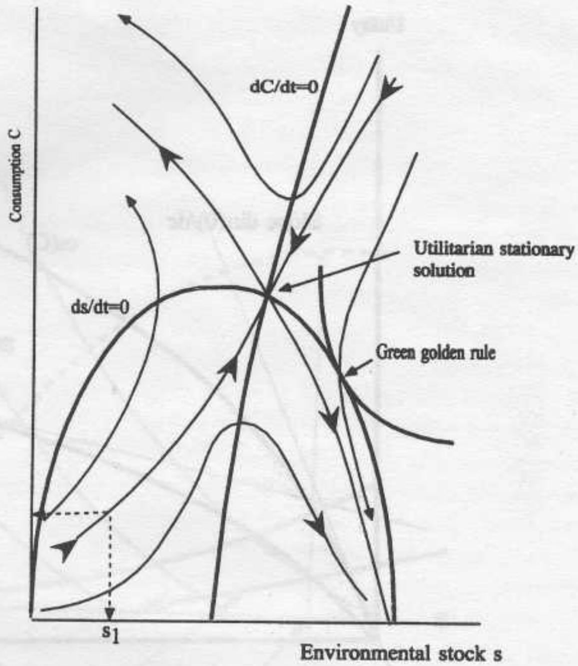


FIGURE 5
Dynamics of the Utilitarian Solution: The Green Golden Rule—the Highest Sustainable Utility Level—is the Point of Tangency Between an Indifference Curve and the Growth Curve.

both cases, because the stock of the resource is an argument of the utility function, the stationary stock exceeds that giving the maximum sustainable yield. This is an obvious consequence of the utility of the stock in its own right.

### *V.B. Exhaustible Resources*

Consider now the rather simpler case of exhaustible resources, in essence an extension of the familiar Hotelling case. For sustainable preference we consider the problem

$$\left.\begin{aligned} &\max \alpha \int_0^\infty u(c_t, s_t) e^{-\delta t}\, dt \\ &\quad + (1-\alpha) \lim_{t\to\infty} u(c_t, s_t),\ \alpha > 0, \\ &\text{subject to } \dot{s}_t = -c_t,\ s_t \geq 0\ \forall t \end{aligned}\right\} \qquad [12]$$

and we contrast this with the equivalent

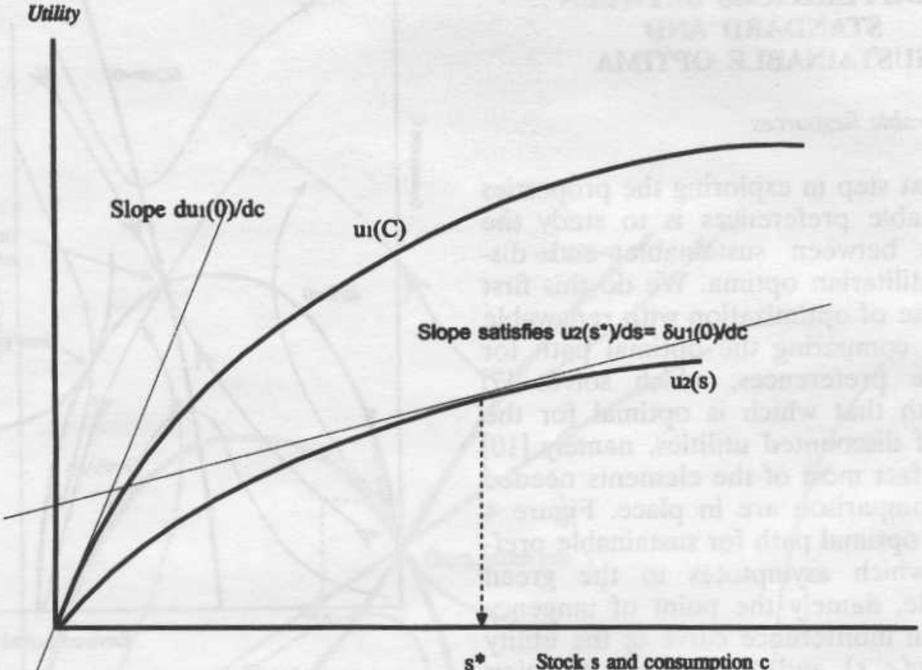


FIGURE 6
Determining the Stationary Stock of the Environmental Asset: The Marginal Utility of the Stock Equals the Marginal Utility of Consumption at Zero Times the Discount Rate. The Stock Rises as the Discount Rate Falls.

discounted utilitarian formulation

$$\left.\begin{aligned}&\max \int_0^\infty u(c_t, s_t)e^{-\delta t}\,dt\\&\text{subject to } \dot{s}_t = -c_t,\, s_t \geq 0\ \forall t.\end{aligned}\right\} \quad [13]$$

In both cases the stock of the resource is an argument of the utility function, so that the resource is in the category of environmental assets such as forests, landscapes, biodiversity, etc., which provide services and value to human society via their stocks as well as via a flow of consumption. The solution to the utilitarian case is summarized in the following theorem, which is illustrated in Figures 6 and 7:

THEOREM 4. *Consider an optimal solution to problem* [12] *when the utility function is additively separable,* $u(c, s) = u_1(c) + u_2(s)$. *A sufficient condition for the optimum to involve the preservation of a positive stock forever is that the marginal utility of consumption at zero is finite,* $u_1'(0) < \infty$, *and that there exists a finite stock level* $s^*$, *the optimal stationary stock, such that* $u_1'(0)\delta = u_2'(s^*)$. *In this case, if the initial stock* $s_0 > s^*$, *then total consumption over time will equal* $s_0 - s^*$: *if* $s_0 \leq s^*$, *then consumption will always be zero and the entire stock will be conserved on an optimal path. If on the other hand the marginal utility of consumption at* $c = 0$ *is infinite, then it will not be optimal to conserve any positive stock level indefinitely.*

The determination of the stock $s^*$ which is conserved forever is illustrated in Figure 6. Figure 7 shows in the space of consumption and the remaining stock the phase diagram for the differential equations which define the conditions necessary for optimal-

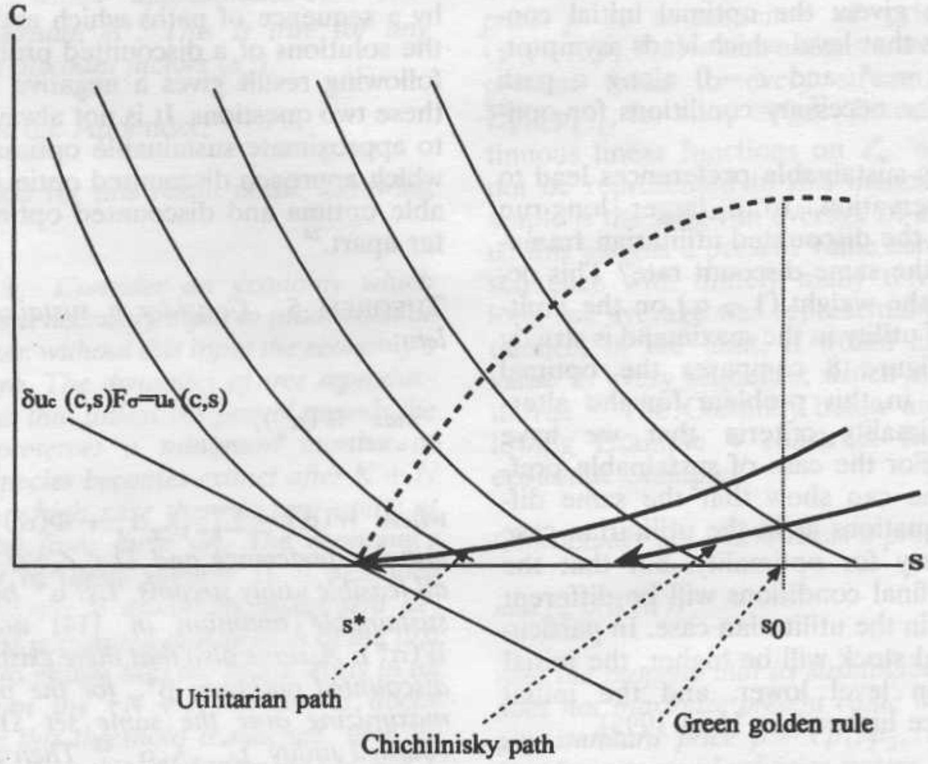


FIGURE 7
The Dynamics of Depletion Paths Optimal According to Alternative Optimality Criteria.

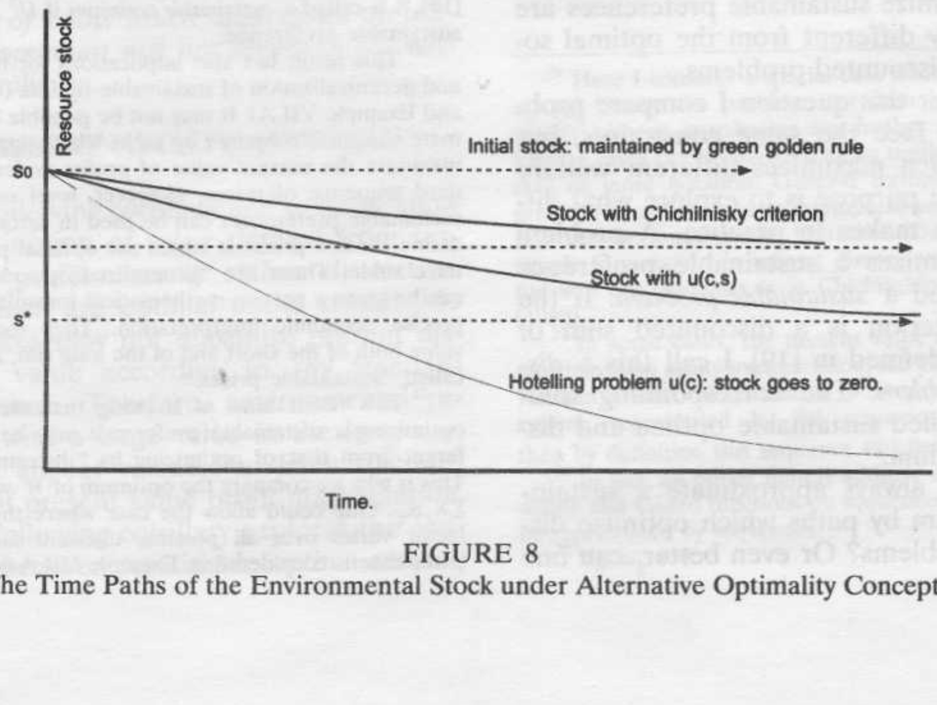


FIGURE 8
The Time Paths of the Environmental Stock under Alternative Optimality Concepts.

ity in the utilitarian case [13]. The initial stock $s_0$ is given: the optimal initial consumption is that level which leads asymptotically to $s = s^*$ and $c = 0$ along a path satisfying the necessary conditions for optimality.

When do sustainable preferences lead to more conservation, to a larger long-run stock, than the discounted utilitarian framework with the same discount rate? This occurs when the weight $(1 - \alpha)$ on the limiting value of utility in the maximand is strictly positive. Figure 8 compares the optimal paths of $s$ in this problem for the alternative optimality criteria that we have examined. For the case of sustainable preferences, one can show that the same differential equations as in the utilitarian case are necessary for optimality, but that the initial and final conditions will be different from those in the utilitarian case. In particular, the final stock will be higher, the initial consumption level lower, and the initial shadow price higher (see Heal 1995).

## VI. SUSTAINABLE OPTIMA THAT ARE FAR FROM DISCOUNTED OPTIMA

We saw that sustainable preferences are substantially different from other welfare criteria which have been used in the literature. It remains however to study how different they are in practice, for example, whether the optimal solutions of problems which maximize sustainable preferences are substantially different from the optimal solutions to discounted problems.

To answer this question I compare problems which face the same constraints, but each of which maximizes different welfare criteria. The purpose is to explore what difference this makes in practice. A problem which maximizes a sustainable preference will be called a *sustainable problem*. If the welfare criterion is a discounted sum of utilities as defined in [19], I call this a *discounted problem*. The corresponding solutions are called sustainable optima and discounted optima.[23]

Can one always approximate a sustainable optimum by paths which optimize discounted problems? Or even better: can one always approximate a sustainable optimum by a sequence of paths which approximates the solutions of a discounted problem? The following result gives a negative answer to these two questions. It is not always possible to approximate sustainable optima by paths which approach discounted optima. Sustainable optima and discounted optima can be far apart.[24]

THEOREM 5. *Consider a sustainable problem*:

$$\max_{\{\alpha : \alpha \in \Omega\}} W(\alpha_g), \qquad [14]$$

*where* $W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g + \Phi(\alpha)$ *is a sustainable preference and* $\Omega \subset \ell_\infty$ *is the set of all feasible utility streams. Let* $\alpha^*$ *be a unique sustainable optimum of* [14] *and* $W^* = W(\alpha^*)$. *Assume also that there exists a unique discounted optimum* $\beta^*$, *for the problem of maximizing over the same set* $\Omega$ *the discounted utility* $\sum_{g=1}^{\infty} \lambda_g \alpha_g$.[25] *Then in general the sustainable optima* $\alpha^*$ *cannot be approximated by a sequence of feasible utility streams*

[23] Formally: let $F$ be a convex and closed subset of feasible paths in a linear space $X$ such as, for example, $X = \ell_\infty$, or $X = R^N$. A vector $\beta \in F$ is called optimal in $F$ if it maximizes the value of a function $U$: $X \to R$. The vector $\beta$ is called a *discounted optimum* if $U$: $\ell_\infty \to R$ is a discounted sum of utilities as defined in [19]; $\beta$ is called a *sustainable optimum* if $U$: $X \to R$ is a sustainable preference.

[24] This result has also implications for the support and decentralization of sustainable optima (Corollary 6 and Example VII.A). It may not be possible to approximate sustainable optima by paths which approximately maximize the present value of profits under any standard sequence of prices. However, level independent sustainable preferences can be used in certain cases to define shadow prices at which the optimal paths maximize value. These are "generalized" prices, but they can be given a precise mathematical formulation and a precise economic interpretation. They measure the value both of the short and of the long run, and can be called "sustainable prices."

[25] This result aims at showing that the result of optimizing a sustainable preference may be quite different from that of optimizing its "discounted" part. This is why we compare the optimum of $W$ with that of $\sum \lambda_g \alpha_g$. One could allow the case where the discount factor varies over all possible discount factors; this latter case is considered in Example VII.A below.

$\{\beta^n\}_{n=1,2,\ldots}$ *which approximates the discounted optimum* $\beta^*$. *This is true for any sequence of discount factors* $\{\lambda_g\}$.

PROOF. In the Appendix.

The intuition for this result is the following example:

EXAMPLE 1. *Consider an economy which uses trees as a necessary input to production or consumption; without this input the economy's utility is zero. The dynamics of tree reproduction requires that unless the first N periods the economy preserves a minimum number of trees, the species becomes extinct after* $K + N$ *periods, in which case there is zero utility at every period from there on. The economy's feasible set of utility streams* $\Omega$ *is described then as follows: a minimum investment denoted* $\epsilon > 0$ *is required during each of the first N periods to ensure that the utility levels in all periods from the* $(N + K)$*-th on, is above zero. Once this threshold is reached, then all utility levels in each period after the* $(N + K)$*-th exceed* $\epsilon$. *Then for every discount factor, there is an N, K for which the sum of discounted utilities is maximized at a path which leads to the eventual elimination of the forest. Instead, for a sustainable preference which gives sufficient weight to the long run the optimum will keep the forest alive and yielding a minimum utility level* $\epsilon$ *forever. Therefore the two optima are apart by at least* $\epsilon$; *any sequence of paths which approaches the discounted optimum will not approach the sustainable solution.*

## VII. SUSTAINABILITY AND VALUE

The following corollary and example show that the notion of value derived from sustainable preferences is rather distinctive. Paths which are optimal under sustainable preferences may not maximize present discounted value according to any standard price system. Therfore, environmental resources with a large value in the long run, may not appear valuable under a standard notion of present value profit maximization.

The following corollary explores the connection between sustainable optima and the maximization of present value.[26] A *standard price p* is a sequence of prices, $p = (p_1, \ldots, p_g, \ldots)$ which assigns a well-defined present value to every stream, $p(\gamma) = \sum_{g=1}^{\infty} p_g \gamma_g$, for all $\gamma \in \ell_\infty$.[27] There exist continuous linear functions on $\ell_\infty$ which cannot be represented in this manner. An example is the long-run average of a sequence $\alpha$: this assigns a present value zero to every sequence with finitely many terms. If the long-run average was representable as a sequence, in the limit, it would assign zero value to every sequence, which is obviously untrue.[28] The Corollary below and the following Example 4 construct two specific economic examples.

COROLLARY 6. *There exists a problem*[29]

$$max_{\alpha \in \Omega}(W(\alpha)) \qquad [15]$$

*with the property that its sustainable optimum does not maximize present value within* $\Omega$ *at any standard price* $p = (p_1, p_2, \ldots)$. *Therefore for any standard price system p there are suboptimal paths which have strictly larger present value than the sustainable optimum* $\alpha^*$, *that is, for all p,* $\exists \beta^* \in \Omega$:

$$\sum_{g=1}^{\infty} p_g \alpha_g^* < \left( Sup_{\beta \in \Omega} \sum_{g=1}^{\infty} p_g \cdot \beta_g \right).$$

*This result is true for any discount factor.*

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[26] Here I consider a special case where the utilities $u_g$ are linear; the problem can then be formulated readily without introducing any further notation. The general cases can be analyzed along similar lines, at the cost of more notation. General formulations of the problem of optima and intertemporal profit maximization can be found in the literature (Debreu 1954); a simple formulation in infinite dimensional spaces that fits well our purposes is in Chichilnisky and Kalman (1980).

[27] In some cases, the present value coincides with intertemporal utility maximization, see Chichilnisky and Kalman (1980). Note that if $p$ is a standard price system, represented by the sequence $(p_1, p_2, \ldots)$, then by definition this sequence satisfies $\sum_{g=1}^{\infty} p_g < \infty$.

[28] In fact, all purely finitely additive measures on $Z$ define real valued functions on sequences which cannot be represented by sequences.

[29] $\forall g$, $u_g \neq 0$.

PROOF. Without loss of generality, consider the case where the utility functions $u_g$ are the identity map. In this case, the set of feasible utility streams $\Omega$ coincides with the set of feasible consumption streams. Furthermore, since the welfare function $W$ defines an independent sustainable preference, the first term of $W$ defines a sequence of standard prices with the desirable properties, namely $p = \{\lambda_g\}_{g=1,2,\ldots}$ defines the present value $\sum_g \lambda_g \alpha_g$ for each sequence of $\ell_\infty$.[30] Theorem 3 showed that in general the value maximizing sequence $\beta^*$ is different from $\alpha^*$. In particular, this implies that the sustainable optimum $\alpha^*$ does not generally maximize value for the standard price system $p$. Since the results of Theorem 5 are true for any sequence of "discount factors" $\{\lambda_g\}_{g=1,2,\ldots}$ satisfying $\forall_g$, $\lambda_g > 0$ and $\sum_{g=1}^{\infty} \lambda_g < \infty$, the corollary follows.

*VII.A. A Sustainable Optimum Which Does Not Maximize Expected Value at Any Standard Price System*

The results of Corollary 6 can be strengthened further by means of another example. Consider a feasible path $\beta \in \ell_\infty$ which maximizes a continuous concave utility function $U$ within a convex set $F \subset \ell_\infty$, but such that at no standard price system $p$ does $\beta$ maximize present value.[31] For $c \in [0, \infty)$ let

$u_t(c) = 2^t c$ for $c \leq 1/2^{2t}$ and

$u_t(c) = 1/2^t$ for $c > 1/2^t$.

Now, for any sequence $c \in \ell_\infty^+$ let $U(c) = \sum_{t=1}^{\infty} u_t(c_t)$, which is well defined, continuous, concave, and increasing of $\ell_\infty^+$. Let $\beta \in \ell_\infty^+$ be defined by

$$\beta_t = 1/2^{2t+1}$$

and let[32]

$$F = U^\beta = \{\lambda \in \ell_\infty : U(\gamma) \geq U(\beta)\};$$

$F$ is a closed convex subset of $\ell_\infty$. Now assume that $p = \{p_t\}_{t=1,2,\ldots}$ is a standard supporting price system for the set $U^\beta$, $p_t \geq 0$, that is, $p \cdot \gamma \geq p \cdot \beta \;\; \forall \gamma \in U^\beta$. By the usual marginal rate of substitution arguments,

$$p_t = p_1 2^{t-1}. \tag{16}$$

I shall show that $p_1$ must be zero, so that the whole sequence $\{p_t\}_{t=1,2,\ldots}$ must be zero. Assume to the contrary that $p_1 \neq 0$. Define $z \in \ell_\infty^+$ by

$$z_t = 1/p_t$$

and $z^n \in \ell_\infty^+$ by

$$z_t^n = \alpha_t^* \text{ if } t \leq n, \text{ and } 0 \text{ otherwise.}$$

Then $\forall n, z \geq z^n$ so that

$$p(z) \geq p(z^n), \tag{17}$$

but

$$\sum_{t=1}^{\infty} p_t z_t^n = n > p(z)$$

for some $n$ sufficiently large, contradicting [17].

The contradiction arises from the assumption that $p_1$ is not zero. Therefore $p_1 = 0$ and by [16] the entire price sequence $p = \{p_t\}_{t=1,2,\ldots}$ is identically zero. It is therefore not possible to support the con-

[30] It can be shown that the value $p(\alpha) = \sum_g \lambda_g \alpha_g$ can be interpreted as the intertemporal profit of the "plan" $\alpha$.

[31] This is from Example 1 in Chichilnisky and Heal (1993, 369), which is reproduced here for the reader's convenience. This example deals with the minimization rather than the optimization of a function over a set, but the results are of course equivalent. The example constructs a feasible set $F \subset \ell_\infty$ which is non-empty, closed and concave, and a continuous concave function $U: \ell_\infty^+ \to R$ which attains a non-zero infimum $U(\beta)$ at $\beta$ in $F$, such that the only sequence of prices $p = \{p_n\}_{n=1,2,\ldots}$ which can support $\beta$ in $F$ is identically zero.

[32] We call this set $U^\beta$ in sympathy with the notation of Chichilnisky and Heal (1993).

cave set $U^{\beta}$ with a non-zero standard price system.[33]

## VIII. CONCLUSIONS

I have defined a set of axioms which capture the idea of sustainability, and characterized the sustainable preferences that they imply. I also analyzed other criteria used in the literature, and found that they do not satisfy my axioms. Discounted utility fails to satisfy the non-dictatorship of the present. This agrees with the viewpoint of many practitioners, who have pointed out the inadequacy of discounted utility for analyzing sustainable growth.[34] Rawlsian and basic needs criteria are insensitive, since they only regard the welfare of the generation which is least well-off. The *overtaking* criterion and its relative the *catching up* criterion are incomplete as orders and cannot be represented by real valued functions. They fail to compare many reasonable alternatives. This decreases their value as tools for decision making. Ramsey's criterion has a similar drawback: it is defined as the integral of the distance to a "bliss" utility level, but this integral is often ill-defined.[35]

The sustainable preferences proposed here and characterized above circumvent all of these problems. From the practical point of view, they satisfy two desirable criteria: they fit our intuition of finite horizon problems, because in important examples they have a turnpike property with respect to equal treatment finite horizon problems. In addition, they fit rather well empirical observations that indicate that people's perceptions of the future imply lower discount rates as time progresses. Important classes by dynamic problems have a solution according to sustainable preferences only if the implied discount rates are decreasing through time.

I also showed that sustainable preferences give rise to optimal solutions which are different from those obtained by discounted optimization criteria. A path which is optimal under a sustainable preference may not be approximated by paths which approximate discounted optima.

The notion of value derived from sustainable preferences is distinctive. Paths which are optimal under sustainable preferences may not maximize value according to any standard price system. Therefore, environmental resources with a large value in the long run, may not appear valuable under a standard notion of profit maximization.

These results may help to disentangle the apparent contradictions in values which were discussed in the beginning of this paper. We noted that governments and international organizations appear seriously concerned about global environmental problems which lie so far into the future that with current discounted utility measures they do not lead to substantial economic loss. The axioms for sustainable preferences proposed here may help resolve this contradiction. Discounted profit maximization and sustainability lead to different value systems. Some trade-offs are possible, but the two values are not the same. The empirical evidence we have today is more in favor of sustainable preferences than discounted utility. Solow (1992) has proposed that sustainability should allow intergenerational trade-offs, but no generation should be favored over any other. This standard is met by sustainable preferences when applied to the "present" and to "future" generations. The long run does matter and so does the short run. Indeed, independent sustainable preferences can define *shadow prices* for sustainable optima, which can be used for project evaluation and for the characterization of optimal solutions. Several of the aims of this paper have there-

[33] Further examples of phenomena related to the results in Theorem 2 and Corollary 1 can be found in Dutta (1991).

[34] For example, Dasgupta and Heal (1979), Broome (1992), Cline (1992).

[35] Hammond (1993) has defined agreeable paths as those which are approximately optimal for any sufficiently long horizon, in the sense that the welfare losses inflicted by considering only finite horizons go to zero as the length of the finite horizon goes to infinity. The criterion is not designed as a complete order but rather as a way of identifying acceptable paths. A similar issue arises with the overtaking criterion, which is ill-defined in many cases.

fore been reached, and several of the questions that we posed have been answered.

But perhaps the results open up at least as many new questions. It remains to understand the concern for the long-run future which is observed in practice, and which appears formalized in the axioms proposed here and their implied preferences. Nobody alive today, not even their heirs, has a stake on the welfare of fifty generations into the future. Yet many humans care about the long-run future of the planet, and the results of this paper indicate that axioms which formalize this concern are not altogether unacceptable. One may then ask: whose welfare do sustainable preferences represent?

Perhaps an answer for this riddle may be found in a wider understanding of humankind as an organism who seeks its overall welfare over time. Such proposals have been advanced in the concepts of a "selfish gene," or, more practically, in Eastern religions which view the unity of humankind as a natural phenomenon. If such unity existed, humankind would make up an unusual organism, one whose parts are widely distributed in space and time and who is lacking a nervous system on which the consciousness of its existence can be based. Perhaps the recent advances in information technology, with their global communications and processing reach, are a glimmer of the emergence of a nervous system from which a global consciousness for humankind could emerge.

## IX. APPENDIX

### *IX.A. Continuity*

In practical terms the continuity of $W$ is the requirement that there should exist a sufficient statistic for inferring the welfare criterion from actual data. This is an expression of the condition that it should be possible to approximate as closely as desired the welfare criterion $W$ by sampling over large enough finite samples of utility streams. Continuity of a sustainable criterion function $W: \ell_\infty \to R$ is not needed in Theorem 1; it is used solely for the characterization in Theorem 2. Continuity is defined in terms of the standard topology of $\ell_\infty$: the norm defined by $\|\alpha\| = \sup_{g=1,2,\ldots} |\alpha(g)|$.

### *IX.B. Independence*

The welfare criterion $W: \ell_\infty \to R$ will be said to give independent trade-offs between generations, and called *independent*, when the marginal rate of substitution between the utilities of two generations $g_1$ and $g_2$ depends only on the identities of the generations, that is, on the numbers $g_1$ and $g_2$, and not on the utility levels of the two generations. Independence of the welfare criterion is not needed in Theorem 1. It is used solely in the characterization of Theorem 2, to allow us to obtain a simple representation of all sustainable preferences. Formally: let $\ell_\infty^*$ be the space of all continuous real valued linear functions on $\ell_\infty$.

DEFINITION 10. *The welfare criterion* $W: \ell_\infty \to R$ *is independent if* $\forall \alpha, \beta \in \ell_\infty$,

$$W(\alpha) = W(\beta) \Leftrightarrow$$

$$\exists \lambda \in \ell_\infty^*, \lambda = \lambda(W), \quad \textit{such that } \lambda(\alpha) = \lambda(\beta).$$

This property has a simple geometric interpretation, which is perhaps easier to visualize in finite dimensions. For example: consider an economy with $n$ goods and two periods. Let $\alpha = (\alpha_1 \alpha_2)$, $\beta = (\beta_1, \beta_2) \in R^2$ denote two feasible utility streams. Then $\alpha$ and $\beta$ are equivalent according to the welfare criterion $W: R^2 \to R$, that is, $W(\alpha) = W(\beta)$, if and only if there exists a number $\mu = \mu(W)$, $\mu > 0$, such that

$$\frac{\alpha_2 - \beta_2}{\alpha_1 - \beta_1} = \mu. \qquad [18]$$

The geometric interpretation of [18] is that the indifference surfaces of $W$ are affine linear subspaces of $R^2$. Level independence implies that the indifference surfaces of the welfare function $W$ are affine hyperplanes in $\ell_\infty$. In particular, $W$ can be represented by a linear function on utility streams, that is, $W(\alpha + \beta) = W(\alpha) + W(\beta)$. Examples of welfare criteria which satisfy this axiom are all time-separable discounted utility functions, any linear real valued non-negative function on $\ell_\infty$, and the welfare criteria in Theorem 2. As already mentioned, this axiom is used to provide a tight representation of sustainable preferences, but is not strictly necessary for the main results.

DEFINITION 11. *A continuous independent sustainable preference is a complete sensitive preference satisfying Axioms 1 and 2 and which is continuous and independent.*

*IX.C. Definition of Previous Welfare Criteria*

To facilitate comparison, this section defines some of the more widely used welfare criteria. A function $W: \ell_\infty \to R$ is called a *discounted sum of utilities* if it is of the form:

$$W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g, \quad \forall \alpha \in \ell_\infty, \qquad [19]$$

where $\forall g$, $\lambda_g \geq 0$ and $\Sigma_{g=1}^{\infty} \lambda_g < \infty$; $\lambda$ is called the discount factor. *Ramsey's welfare criterion* (Ramsey 1928) ranks a utility stream $\alpha = \{\alpha_g\}_{g=1,2,\ldots} \in \ell_\infty$ above another $\beta = \{\beta_g\}_{g=1,2,\ldots} \in \ell_\infty$ if the utility stream $\alpha$ is "closer" to the bliss path, namely to the sequence $\zeta = \{1, 1, \ldots, 1, \ldots\}$, than is the sequence $\beta$. Formally:

$$\sum_{g=1}^{\infty} (1 - \alpha_g) \leq \sum_{g=1}^{\infty} (1 - \beta_g). \qquad [20]$$

*A Rawlsian rule* (Rawls 1971) ranks two utility streams according to which has a higher infimum value of utility for all generations.[36] This is a natural extension of the criterion proposed initially by Rawls (1971). Formally: a utility stream $\alpha$ is preferred to another $\beta$ if

$$\inf\{\alpha_g\}_{g=1,2,\ldots} > \inf\{\beta_g\}_{g=1,2,\ldots} \qquad [21]$$

The criterion of *satisfaction of basic needs* introduced in Chichilnisky (1977a) ranks a utility stream $\alpha$ over another $\beta$ if the time required to meet basic needs is shorter in $\alpha$ than in $\beta$. Formally:

$$T(\alpha) \leq T(\beta), \qquad [22]$$

where $T(\alpha) = \min\{t: \alpha_g \geq b \ \forall_g \geq t\}$, for a given $b$ which represents basic needs. The *overtaking criterion* (von Weizäcker 1967) ranks a utility stream $\alpha$ over another $\beta$ if $\alpha$ eventually leads to a permanently higher level of aggregate utility than does $\beta$. Formally: $\alpha$ is preferred to $\beta$ if $\exists N$:

$$\forall M > N, \quad \sum_{g=1}^{M} \alpha_g \geq \sum_{g=1}^{M} \beta_g. \qquad [23]$$

The *long-run average* criterion can be defined in our context as follows: a utility stream $\alpha$ is preferred to another $\beta$ if in average terms, the long-run aggregate utility[37] achieved by $\alpha$ is larger than achieved by $\beta$. Formally: $\exists N, K > 0$:

$$\frac{1}{T}\left(\sum_{g=M}^{T+M} \alpha_g\right) \geq \frac{1}{T}\left(\sum_{g=M}^{T+M} \beta_g\right),$$
$$\forall T > N \text{ and } M > K. \qquad [24]$$

*IX.D. Countable and Finitely Additive Measures*

DEFINITION 12. *Let $(S, \Sigma)$ denote the field of all subsets of a set S with the operations of unions and intersections of sets. A real valued, bounded additive set function on $(S, \Sigma)$ is one which assigns a real value to each element of $(S, \Sigma)$, and assigns the sum of the values to the union of two disjoint sets.*

DEFINITION 13. *A real valued bounded additive set function is called countably additive if it assigns the countable sum of the values to a countable union of disjoint sets.*

EXAMPLE 2. *Probability measures on the real numbers, R, or on the integers Z, are typical examples of such countably additive functions. Any sequence of positive real numbers $\{\lambda_g\}_{g=1,2,\ldots}$ such that $\Sigma_{g=1}^{\infty} \lambda_g < \infty$ defines a countably additive measure $\mu$ on the integers Z, by the rule*

$$\mu(A) = \sum_{g \in A} \lambda_g, \forall A \subset Z.$$

DEFINITION 14. *A real valued bounded additive set function $\varphi$ on $(S, \Sigma)$ is called purely finitely additive (see Yosida and Hewitt 1952) if whenever a countably additive function $v$ satisfies:*

$$\forall A \in (S, \Sigma), \quad v(A) \leq \varphi(A), \quad \text{then}$$
$$v(A) = 0 \quad \forall A \in (S, \Sigma).$$

This means that the only countably additive measure which is absolutely continuous with respect

[36] Related Rawlsian rules are discussed in Asheim (1988).

[37] This is only one of the possible definitions of long-run averages. For other related definitions with similar properties see Dutta (1991).

to a purely finitely additive measure, is the measure which is identically zero.

EXAMPLE 3. *Any real valued linear function* $V$: $\ell_\infty \to R$ *defines a bounded additive function* $\hat{V}$ *on the field* $(Z, \Sigma)$ *of subsets of the integers* $Z$ *as follows*:

$$\forall A \subset Z, \; \hat{V}(A) = V(\alpha^A) \qquad [25]$$

*where* $\alpha^A$ *is the "characteristic function" of the set* $A$, *namely the sequence defined by*

$$\alpha^A = \{\alpha_g^A\}_{g=1,2,\ldots} \quad \text{such that}$$

$$\alpha_g^A = 1 \quad \text{if } g \in A \quad \text{and} \quad \alpha_g^A = 0 \quad \text{otherwise.} \qquad [26]$$

EXAMPLE 4. *Typical purely finitely additive set functions on the field of all subsets of the integers,* $(Z, \Sigma)$, *are the* lim inf *function on* $\ell_\infty$, *defined for each* $\alpha \in \ell_\infty$ *by*

$$\liminf(\alpha) = \liminf_{g=1,2,\ldots} \{\alpha_g\}. \qquad [27]$$

*Recall that the* lim inf *of a sequence is the infimum of the set of points of accumulation of the sequence. The "long-run averages" function is another example: it is defined for each* $\alpha \in \ell_\infty$ *by*

$$\lim_{K,N\to\infty} \left( \frac{1}{K} \sum_{g=N}^{K+N} \alpha_g \right). \qquad [28]$$

It is worth noting that a purely finitely additive set function $\phi$ on the field of subsets of the integers $(Z, \Sigma)$ cannot be represented by a sequence of real numbers in the sense that there exists no sequence of positive real numbers, $\lambda = \{\lambda_n\}$ which defines $\phi$, that is, there is for no $\lambda$ such that

$$\forall A \subset Z, \; \phi(A) = \sum_{n \in A} \lambda_n.$$

For example the lim inf: $\ell_\infty \to R$, defines a purely finitely additive set function on the integers which is not representable by a sequence of real numbers.

### IX.E. *Proof of Theorem 1*

PROOF. To establish the existence of a sustainable preference $W$: $\ell_\infty \to R$, it suffices to exhibit a function $W$: $\ell_\infty \to R$ satisfying the two axioms. For any $\alpha \in \ell_\infty$ consider

$$W(\alpha) = \Sigma_{g=1}^{\infty} \delta^g \alpha_g + \left[\liminf\{\alpha_g\}_{g=1,2,\ldots}\right],$$

with $0 < \delta < 1$.

$W$ satisfies the axioms because it is a well-defined, non-negative, increasing function on $\ell_\infty$; it is not a dictatorship of the present (Axiom 1) because its second term makes it sensitive to changes in the "tails" of sequences; it is not a dictatorship of the future (Axiom 2) because its first term makes it sensitive to changes in "cutoffs" of sequences.

The next task is to show that the following welfare criteria do not define sustainable preferences: (a) Ramsey's criterion, (b) the overtaking criterion, (c) the sum of discounted utilities, (d) lim inf, (e) long-run averages (f) Rawlsian criteria, and (g) basic needs. The Ramsey's criterion defined in [20] fails because it is not a well-defined real valued function on all of $\ell_\infty$, and cannot therefore define a complete order on $\ell_\infty$. To see this it suffices to consider any sequence $\alpha \in \ell_\infty$ for which the sum in [20] does not converge. For example, let

$$\alpha = \{\alpha_g\}_{g=1,2,\ldots} \quad \text{where } \forall g, \; \alpha_g = (g-1)/g.$$

Then $\alpha_g \to 1$ so that the sequence approaches the "bliss" consumption path $\beta = (1, 1, \ldots, 1, \ldots)$. The ranking of $\alpha$ is obtained by the sum of the distance between $\alpha$ and the bliss path $\beta$. Since $\lim_{N\to\infty} \Sigma_{g=1}^{N}(1 - \alpha_g) = \lim_{N\to\infty} \Sigma_{g=1}^{N} 1/g$ does not converge, Ramsey's welfare criterion does not define a sustainable preference.

The overtaking criterion defined in [23] is not a well-defined function of $\ell_\infty$, since it cannot rank those pairs of utility streams $\alpha, \beta \in \ell_\infty$ in which neither $\alpha$ overtakes $\beta$, nor $\beta$ overtakes $\alpha$. Figure 2 exhibits a typical pair of utility streams which the overtaking criterion fails to rank.

The long-run averages criterion defined in [24] and the lim inf criterion defined in [27] fail on the grounds that neither satisfies Axiom 2; both are dictatorships of the future. Finally any discounted utility criterion of the form

$$W(\alpha) = \sum_{g=1}^{\infty} \alpha_g \lambda_g,$$

$$\text{where } \forall_g, \lambda_g > 0 \text{ and } \sum_{g=1}^{\infty} \lambda_g < \infty$$

is a dictatorship of the present, and therefore fails to satisfy Axiom 1. This is because

$$\forall \gamma \in \ell_\infty \text{ s.t. } \sup_{g=1,2,\ldots} (\gamma_g) \leq 1, \text{ and } \forall \epsilon > 0,$$

$$\exists N > 0, \ N = N(\epsilon): \sum_{g=N}^{\infty} \gamma_g < \epsilon, \qquad [29]$$

and therefore, since

$$W(\alpha) > W(\beta) \Rightarrow \exists \epsilon > 0: W(\alpha) - W(\beta) > 3\epsilon,$$

then by [29]

$$\exists N > 0 \text{ such that } \forall \sigma, \gamma \in \Omega,$$

$$W(\alpha^K, \sigma_K) > W(\alpha^K, \gamma_K), \quad \forall K > N.$$

The function $W$ thus satisfies the first part of the definition of a dictatorship of the present, that is,

$$W(\alpha) > W(\beta) \Rightarrow$$

$$\exists N, \quad N = N(\alpha, \beta): \forall \gamma, \sigma \in \ell_\infty$$

$$\text{with } \|\gamma\| \leq 1 \text{ and } \|\sigma\| \leq 1,$$

$$W(\alpha^K, \gamma_K) > W(\beta^K, \sigma_K), \quad \forall K > N.$$

The reciprocal part of the definition of dictatorship of the present is immediately satisfied, since if $\forall \sigma, \gamma \in \ell_\infty$ such that $\|\alpha\| \leq 1$, $\|\beta\| \leq 1$, $W(\alpha^K, \sigma_K) > W(\alpha^K, \gamma_K)$, and obviously this implies $W(\alpha) > W(\beta)$. Therefore $W$ is a dictatorship of the present and violates Axiom 1.

Finally the Rawlsian welfare criterion and the criterion of satisfaction of basic needs do not define independent sustainable preferences: the Rawlsian criterion defined in [21] fails because it is not sensitive to the welfare of many generations: only to that of the less favored generation. Basic needs has the same drawback.

### *IX.E. Proof of Theorem 2*

PROOF. Consider a continuous independent sustainable preference. It must satisfy Axioms 1 and 2. There exists a utility representation for the welfare criterion $W: \ell_\infty \to R$, defining a non-negative, continuous linear functional on $\ell_\infty$. As seen above in Example 3, [25] and [26], such a function defines a non-negative, bounded, additive set function denoted $\hat{W}$ on the field of subsets of the integers $Z, (Z, \Sigma)$.

The representation theorem of Yosida and Hewitt (Yosida 1974; Yosida and Hewitt 1952) establishes that every non-negative, bounded, additive set function on $(S, \Sigma)$, the field of subsets $\Sigma$ of a set $S$, can be decomposed into the sum of a non-negative measure $\mu_1$ and a purely finitely additive, non-negative set function $\mu_2$ on $(S, \Sigma)$. It follows from this theorem that $\hat{W}$ can be represented as the sum of a countably additive measure $\mu_1$, and a purely finitely additive measure on the integers $Z$. It is immediate to verify that this is the representation in [5]. To complete the characterization of an independent sustainable preference it suffices now to show that neither $\lambda$ nor $\phi$ are identically zero in [5]. This follows from Axioms 1 and 2: we saw above that discounted utility is a dictatorship of the present, so that if $\phi \equiv 0$, then $W$ would be a dictatorship of the present, contradicting Axiom 1. If on the other hand $\lambda \equiv 0$, then $W$ would be a dictatorship of the future because all purely finitely additive measures are, by definition, dictatorships of the future, contradicting Axiom 2. Therefore neither $\lambda$ nor $\phi$ can be identically zero. This completes the proof of the theorem.

### *IX.F. Proof of Theorem 5*

The statement of Theorem 5 is: Consider a sustainable optimum growth problem

$$\max_{\alpha \in \Omega} W(\alpha_g),$$

$$\text{where } \alpha_g = \{u_g(x_g)\}_{g=1,2,\ldots} \in \Omega \subset \ell_\infty, \quad [30]$$

where $\Omega$ is the set of all feasible utility streams and $W$ is an independent sustainable preference. By Theorem 2 $W$ must be of the form:

$$W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g + \phi(\alpha), \quad \forall \alpha \in \ell_\infty, \qquad [31]$$

where $g$, $\lambda_g > 0$, $\Sigma_{g=1}^{\infty} \lambda_g < \infty$, and $\phi \neq 0$ is a purely finitely additive independent measure on $Z$. Assume that there exists a unique solution to problem [30], denoted $\alpha^*$ and called a sustainable optimum, with welfare value $W^* = W(\alpha^*)$. Assume also that there exists a unique solution, denoted $\beta^*$ and called a discounted optimum, for the problem of maximizing over the same set $\Omega$ the discounted utility

$$\max_{\alpha \in \Omega} (U(\alpha)) \quad \text{where } U(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g, \qquad [32]$$

which is the first term defining the preference $W$ in [31]. Then in general the sustainable optima $\alpha^*$ cannot be approximated by a sequence of feasible utility streams $\{\beta^N\}_{n=1,2,\ldots}$ which approximates the discounted optimum $\beta^*$, that is, for all such sequences

$$\lim_{n \to \infty} \left( \sum_{g=1}^{\infty} \lambda_g \beta^n \right) \neq \max_{\gamma \in \Omega} \left( \sum_{g=1}^{\infty} \lambda_g \gamma \right).$$

This is true for any sequence of "discount factors" $\{\lambda_g\}_{g=1,2,\ldots}$ satisfying $\forall g,\ \lambda_g > 0$ and $\sum_{g=1}^{\infty} \lambda_g < \infty$.

PROOF. Define a family of optimal growth problems, each with a welfare function of the form [31], and each having a feasible set $\Omega \subset \ell_\infty$, all satisfying the conditions of the theorem. For each problem in this family, the optimum $\alpha^*$ cannot be approximated by a sequence which approximates the optima $\beta^*$ of discounted utility functions of the form [32]. This is true for any discount factors $\lambda$: $Z \to R$ which satisfy $\forall g,\ \lambda_g > 0,\ \sum_{g=1}^{\infty} \lambda_g < \infty$. The set of feasible utility streams $\Omega$ is as follows:

$$\Omega = \{ \alpha \in \ell_\infty^+ : \alpha = \{\alpha_g\}_{g=1,2,\ldots},\ \sup_g(\alpha_g) \leq 1$$

and $\exists \epsilon > 0$, and integers $N$ and $K$, such that

$$\text{if } \alpha_g \geq \epsilon\ \forall_g \leq N \quad \text{then } \alpha_g = 0\ \forall g > K + N,$$

while if $\alpha_g < \epsilon\ \forall g \leq N$, then

$$\alpha_g \geq \epsilon, \forall g > K + N\}.$$

Each set of parameters $\epsilon$, $N$, and $K$ define a different feasible set of utility streams $\Omega$. If the welfare function $W$ is a discounted utility of the form [32], then there exists $\epsilon$, $N$, and $K$ such that the discounted optimum $\beta^* = \{\beta_g^*\}_{g=1,2,\ldots} \in \Omega$ satisfies

$$\beta_g^* = 1 \quad \text{for } g \leq N + K, \quad \text{and}$$

$$\beta_g^* = 0 \quad \text{for } g > K + N.$$

The sustainable optima $\alpha^*$ is quite different when in the definition of $W$, [31], the purely finitely independent measure $\phi$ has most of the "weight," that is, when $\theta \sim 0$. Indeed, for $\theta \sim 0$, the sustainable optimum $\alpha^*$ satisfies

$$\alpha_g^* \geq \epsilon \quad \text{for } g > K + N.$$

Since both $\alpha^*$ and $\beta^*$ are unique, and

$$\|\alpha^* - \beta^*\| \geq \epsilon > 0,$$

it is clear that a sequence $\{\beta^n\}$ which approaches $\beta^*$ cannot approach also $\alpha^*$. This completes the proof of the theorem.

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