ISSN 0002-9920

Notices

of the American Mathematical Society

March 1996

Volume 43, Number 3

Fischer Black: The Mathematics of Uncertainty

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"The mystery of brilliant productivity will always be the posing of new questions, the anticipation of new theorems that make accessible valuable results and connections. Without the creation of new viewpoints, without the statement of new aims, mathematics would soon exhaust itself in the rigor of logical proofs and begin to stagnate as its substance vanishes. Thus, mathematics has been most advanced by those who distinguished themselves by intuition rather than by rigorous proofs".[1] Fischer Black fits this description well. He was an idea man given to lively debates and to unusual and often unpopular scientific views, whose curiosity led him to create new theoretical connections and to pursue their practical applications in the business world. Born in Washington in 1938, he graduated from Harvard College in physics in 1959 and obtained a Ph.D. in applied mathematics at Harvard in 1964. Thereafter Fischer Black followed his own star. In a rather unconventional way he mixed academia with business, seeking to understand the problems which interested him most, typically in financial markets. His colleagues at Goldman Sachs, the firm where he worked from 1984 until his untimely death,[2] recall "his sense of what's important". This, at the end of the day, is what defines good science. Solving yesterday's famous open theorems is useful and well rewarded. But finding the new paths that matter is essential. Fischer Black had the skill and the judgement to do the latter.

A somewhat aloof and quiet man, he was nevertheless given to strong opinions and frank critical evaluations. His death led to an outpouring of interest from a variety of sources, a testimony of the many different paths that he traveled during his life. In the winter of 1995 several financial journals published articles following his death. Popular publications in economics, such as *The Economist*, wrote also about his contributions to everyday business practice. Black kept working virtually until the last day of his life. His book *Exploring General Equilibrium*, published a few months ago, grapples with issues that involve a mathematical formulation of market behavior and is at the frontier of economics.

Fischer Black married three times and was the father of five children and two stepchildren. He changed jobs several times. Starting as a management consultant in computer science at Bolt, Beranek and Newman for a year, where he worked on information for libraries and hospitals, he then moved to the consulting firm Arthur D. Little (ADL), where he was influenced by colleagues working on institutional and theoretical problems in finance. He left to found his own

[1] *Hermann Weyl reproduces this quote from Felix Klein's lectures on the history of mathematics in his* Unterrichtsblatter fur Mathematik und Naturwissenschaften *38 (1932), 177–188*.

[2] *Fischer Black died of cancer August 30, 1995, at his home in New Canaan, Connecticut, at the age of 57.*

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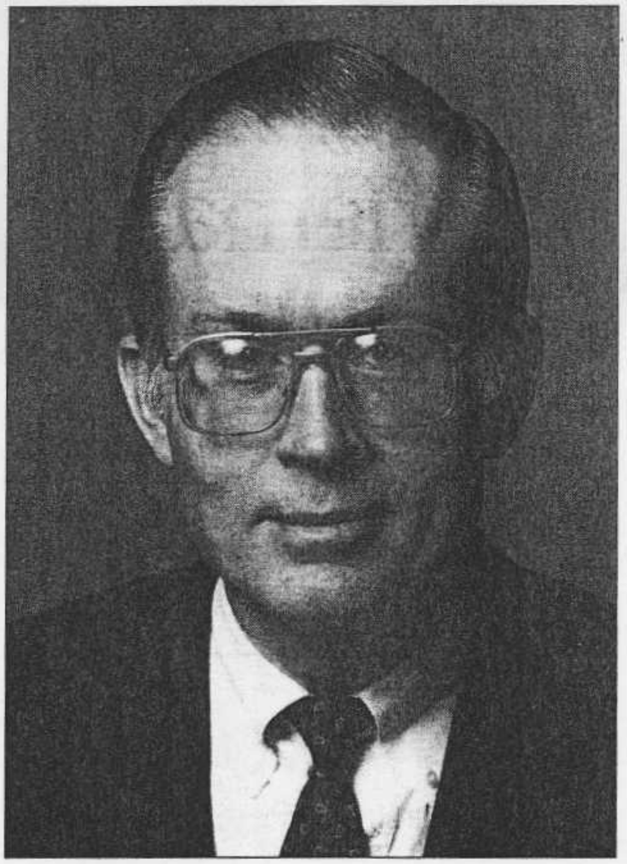

Fischer Black

consulting firm, Associates in Finance, in 1969. In 1971-72 he became a Ford Visiting Professor at the University of Chicago, joining the faculty as a professor in the Graduate School of Business thereafter. In 1975 he joined the faculty of the Sloan School of Management at MIT. He remained at MIT until 1984, when he shifted jobs once again, this time out of academia and back into the business world, to join Goldman Sachs & Company, where he became a partner and worked until his death. Fischer Black was president of the American Finance Association in 1985, was selected Financial Engineer of the Year in 1994, and received the Graham and Dodd Award for the best published paper in *The Financial Analysis Journal* four times.[3] Fischer Black is perhaps best known for his work with Myron Scholes in developing a mathematical model for pricing securities called options. He met Scholes through Michael Jensen, with whom he worked on a consulting project to evaluate mutual funds. Scholes introduced Black to Robert Merton, and the three worked closely on the problem of valuing financial instruments.

Fischer Black's most quoted paper is on the Black-Scholes option pricing model. This paper, "The Pricing of Options and Corporate Liabilities", finds a formula for pricing an option. An option is a security giving the right, but not the obligation, to buy or sell an asset, subject to certain conditions, within a specified period of time. This paper is much quoted and used by theorists and businesspeople alike. Yet, as is the case with many other classic papers in economics, Black had much difficulty in getting his paper accepted for publication. The paper was rejected by several leading journals, appearing finally in 1973 in the *Journal of Political Economy*. A recent piece by Merton and Scholes in *The Journal of Finance* reports that the difficulties of publication were such that the empirical tests of their model appeared in print a full year earlier in the unrefereed annual sessions volume of *The Journal of Finance*.

Black's 1973 paper provides a pricing formula for options which is based on a classic economic principle, *no-arbitrage*. This principle establishes that as long as advantageous trading opportunities exist, the trading activity will not be extinguished. An arbitrage opportunity exists, for example, if a security can be bought in New York at one price and sold at a slightly higher price in London. This price difference creates a riskless opportunity for profits, and trading will not cease until the two prices in New York and in London are brought into line. There is also an arbitrage opportunity if two portfolios that are equivalent to each other, in the sense that they help hedge risks in the same way, can be bought at different prices. Everyone will sell the more expensive portfolio to buy the cheaper one. Trading activity should wipe out such opportunities. At a rest point of the trading activity, called a market "equilibrium", there must be no such arbitrage opportunities. The equilibrium of the trading system leads to the determination of prices, formally through the use of diffusion processes and simple stochastic calculus. This in a nutshell is the concept behind the Black-Scholes formula.

The concept of no-arbitrage pricing is not new, but Black and Scholes gave it a new use. There is a long and distinguished tradition of using no-arbitrage techniques to price assets. In 1931 Harold Hotelling, a highly original mathematical economist at Columbia University, used this method to explain why the prices of exhaustible resources, such as petroleum, must increase at the same rate as the rate of interest. Hotelling's finding[4] was controversial but rests on a sound and now well-accepted fact: that natural resources are assets as much as any other financial assets. People will trade them until their value and their returns match those of other available opportunities.

[3] *A complete list of Fischer Black's publications is provided in an article by Robert Merton and Myron Scholes in the December 1995 issue of* The Journal of Finance.

[4] *H. Hotelling,* The economics of exhaustible resources, *Journal of Political Economy (1931).*

[5] *K. Arrow,* Optimal capital policy with irreversible investment, *Value, Capital and Growth, Papers in Honour of Sir John Hicks (J. Wolfe, ed.), Edinburgh University Press, 1968; K. Arrow and A. Fisher,* Environmental preservation, uncertainty and irreversibility, *Quarterly Journal of Economics (1974); and C. Henry,* Investment decisions under uncertainty: The irreversibility effect, *American Economic Review (1974).*

The connection between the pricing of options and natural resources was also made clear by several other prominent economists—Kenneth Arrow, Anthony Fischer, and Claude Henry—in papers published in 1968 and 1974.[5] Options are valuable, they said, because they allow us to wait until more information is revealed before making a decision. The option gives one the opportunity to buy an asset later at an agreed price and thus postpones the decision to buy until one has more information. In summary, the option allows one to postpone potentially irreversible decisions until more information becomes available. Thus the pricing of an option encodes the value of information.

This issue has become germane in recent years because of concern over possible irreversible long-term environmental changes such as ozone depletion and global warming. We do not know for sure the effects of global emissions of carbon dioxide and ozone, but they could be irreversible: for example, damage such as global climate change—global warming or, at the other extreme, even a new Ice Age. Uncertainty arises due to our inability to predict the effects of emissions accurately.[6] If future damages are uncertain and irreversible, there is value in keeping our options open. It is worth undertaking certain costs in order to keep the current climate pattern when the alternative could be a once-and-for-all irreversible global climate change. The option value is the value of forestalling the point of no return. A formula computing this value tells us how much it is worth to invest in decreasing ozone and carbon emissions today—in other words, the value of keeping our options open.

Hotelling's no-arbitrage pricing of exhaustible resources does not, however, concentrate on uncertainty, while Black and Scholes focus precisely on that. Hotelling's formula is based on the ability to compare the value of oil in the ground with that of a bond paying a fixed rate of return. It has, however, the main element that Black and Scholes focus on in their article: they want a formula that gives a price without depending on the traders' preferences or expectations or other unobservable variables. As pointed out by Black and Scholes, several other economists, such as Sprenkle (1961), Ayres (1963), Boness (1964), Samuelson (1965), Baumol, Malkiel, and Quandt (1966), and Chen (1970), all produced valuation formulas of the same general form but dependent on unknown parameters. The contribution of Black and Scholes is to observe that in equilibrium, when trading has achieved a rest point, the expected return of a hedged position must be equal to that of a riskless asset. They show that this equilibrium condition can be used to derive a theoretical valuation formula. Their formula depends on knowing a short-term interest rate and assumes that the stock price follows a random walk in continuous time with a variance proportional to the square of the stock price. This means that the distribution of stock prices at the end of any finite period is log normal and the variance of the log return is constant. In other words, while there are assumptions and unknown parameters, these depend not on individual characteristics such as expectations but rather on aggregate market behavior on which better observations are possible.[7] The timing of its discovery had a lot to do with the importance that Fischer Black's formula achieved. The world's first options exchange opened in Chicago the same year in which their paper appeared—1973. To offer a perspective on this matter, a few years before, *The Wall Street Journal* had routinely refused to accept advertisements for options trading as an intrinsically shady business. The instruments introduced in Chicago in the 1970s spawned an enormous global trade that led to hedging as much as speculation. Derivatives, the technical term for securities that depend on the value of other assets, have since then flourished in volume and variety. The outstanding value of derivatives at the beginning of 1995 was approximately $20 trillion.

He was an idea man given to lively debates

The importance attached to financial markets in this article may be a source of concern for those who regard mathematical finance as the mathematics of greed. But this would not be

[6]*This can be called scientific uncertainty and can be hedged under certain conditions by financial instruments whose payoffs are contingent on the frequency of unfavourable events, G. Chichilnisky and G. Heal,* Global environmental risks, *Journal of Economics Perspectives (1983).*

[7]*A detailed story of how the formula for valuing options was created is presented in "The Universal Financial Device", a chapter in* Capital Ideas, *P. Bernstein, Free Press, New York, 1992.*

[8]*G. Chichilnisky,* Global environmental risks, *op. cit., and* Catastrophe futures, *Best Review, February 1996. Correlated risks are, by definition, risks which affect most of the population at once. The law of large numbers works by allowing a relatively safe prediction of the proportion of the population that will be affected. Knowing this relatively safe proportion, insurance schemes derive actuarial tables to determine the incidence of risks and to cost the price of selling insurance to different parts of the population. Insurance redistributes the costs between those who suffer the damage and those who do not. When everyone is affected at once, this scheme cannot work.*

an accurate perspective. Financial innovation, namely, the creation of new ways to hedge uncertainty, is often a socially valuable activity. For example, one of the main sources of uncertainty today is the global environment. This is driven by the environmental changes that humans are causing to the atmosphere of the planet and to its climate. Climate change is a new aspect of one of the oldest risks known to humans: weather risks. Today financial markets are developing new ways of coping with climate risks to hedge the human costs of catastrophes such as hurricanes, floods, and earthquakes, which have led to record losses in recent years. The Chicago Board of Trade introduced in 1992 financial instruments called "catastrophe futures". These can be used to complement reinsurance when the risks are heavily correlated, as catastrophic risks often are, and therefore the law of large numbers does not work properly.[8] The mathematics of uncertainty is as fascinating and useful as Fischer Black knew it to be. New issues have arisen which open up new avenues of thought and require the development of new mathematical tools. Risks that are endogenously determined by human activity, which I call *endogenous uncertainty*,[9] open up new and challenging problems in mathematics, economics, and finance. Endogenous risks are those which are caused not solely by nature's moves but also by human actions.[10] Endogenous risks require a different treatment. The pricing of new financial instruments that induce changes in the risk profile of existing instruments, such as derivatives can do, is still an open area. Such instruments define endogenous risks. The formula proposed by Black and Scholes does not cover those instruments because arbitrage cannot be used to deduce prices of instruments that are not fully comparable with others. These new problems are practical in nature, and their solutions demand imagination and new mathematical tools. They require the unusual mix of skills and curiosity that was Fischer Black's trademark.

[9] *G. Chichilnisky,* Global environmental risks, *op. cit., and* Markets with endogenous uncertainty: Theory and policy, *Theory and Decision (1996).*

[10] *Earthquakes are exogenous risks. But the risk of global warming, if this is caused in part by humans' emission of carbon dioxide into the atmosphere of the planet, is an endogenous risk: it is a risk caused by the functioning of the economy. So is the risk of a stock market crash.*