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Energy-Capital Substitution: A General Equilibrium Analysis

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We consider an economy which imports energy from a monopolistic price-setter. The domestic general equilibrium of this economy adjusts in response to the price of energy. We define the *total cross price elasticity* of demand between energy and capital as the cross price elasticity across general equilibria of the economy, as the equilibrium changes in response to energy price changes. This corresponds to the price elasticity given by a total demand curve, and incorporates adjustments on both supply and demand sides. It is shown that whether this total elasticity implies energy-capital complementarity or substitutability depends upon the parameters of the model and the price of energy: for a given model, there may be a change from substitutability to complementarity as the price of energy rises. This framework offers an additional way of reconciling apparently conflicting findings on energy-capital complementarity and substitutability: an earlier suggestion was made by Berndt and Wood (1979). It is a natural extension of the general equilibrium approach initiated by Hogan (1977).

Introduction

The question of whether capital and energy are complements or substitutes is one that has attracted a great deal of attention in the last decade. For a world with only a finite stock of low-cost energy, its importance is obvious. The long-run growth potential of the economy depends crucially on the magnitude of the capital-energy substitution elasticity (see Dasgupta and Heal (1979), Chapter 6); consequently a variety of policy measures also hinge upon this.

Unfortunately, the question, 'Are capital and energy complements or substitutes?' is not an easy one to answer. Indeed, the difficulty is compounded, as we shall show below, by the fact that there are at least three different ways of posing it. There have been some attempts to see whether basic scientific and engineering principles can throw light on the issue: Berry, Heal and Salamon (1978) investigate the implications of thermodynamic parameters of the production functions, but also on parameters of the demand side and on the price of energy.

This leads to a framework within which one may obtain differing econometric results about the complementarity of substitutability of energy and capital, even if all economies have identical production technologies. These variations may arise from differences in the price of energy (Europeans may face higher energy prices than Americans), or from differences in demand conditions (Europeans may be more willing to shift out of energy-intensive consumption patterns). Of course, as in the earlier approaches, differences in technologies could still cause international variations in the econometric findings. However, the fact that virtually identical technologies are available in most OECD countries makes this explanation less appealing than the alternatives since demand conditions and price regimes clearly do vary across countries.

Like the works cited above, we consider an economy which is a price-taker in the international energy market and investigate how its general equilibrium changes with variations in the price of energy. If the price of energy changes from p_0 to p_0 the general equilibrium will alter and, with it, the amount of capital used. We define the total cross price elasticity as the ratio of the proportional response of capital used to the proportional change in the energy price across the general equilibria.

Our method could in principle be used with any general equilibrium specification of an energy-using economy, though in the following sections we shall adopt a simple two-sector three-good general equilibrium model developed by Chichilnisky (1981) for studying the impact of oil price changes on an oil-using economy. The model is complex enough to illustrate the concept of total elasticity and its dependence upon parameter and price regimes, yet still simple enough to be tractable. This model is presented in the next section, where we also use it to analyse the variation of capital demanded with energy prices across equilibria, establishing that substitution characteristics vary with the price of energy.

Intuitively, this is different from studying the gross elasticity as defined by Berndt and Wood (1979) mainly because it takes account of the impact of changes in the relative prices of goods on the composition of demand. It could be the case, for example, that capital and energy are used in fixed proportions in all industries, so that at the micro level there is no possibility of substituting capital for energy. However, when the price of energy rises, the relative prices of energy-intensive goods increase, and consumer demand shifts from these to goods using energy and capital in a lower ratio. Consequently capital-intensive industries expand, energy-intensive industries contract, and overall more capital and less energy are employed. Substitution has occurred through demand shifts as a result of relative price changes. This is at least reminiscent of the argument, originally due to Houthaker (1955), that even though every firm in an economy has a fixed proportions production function, the economy as a whole may behave as if it had a Cobb-Douglas production function. It is also a generalization of the analysis of Akerlof and Burmeister (1970) of substitution in a general equilibrium framework. This is a phenomenon which a one-sector model clearly cannot capture, yet which may be important at the aggregate level.

The Model

The model we shall use to illustrate our concept of total elasticity and its dependence on price and parameter regimes is that developed by Chichilnisky (1981). It is a two-sector model with three productive factors: capital, labour and oil. Within each sector, production functions display fixed input proportions, so that no substitution is possible. However, the two sectors differ in their factor intensities, so that changes in relative factor prices lead to changes in relative goods prices and hence to substitution on the demand side. This in turn leads to changes in the proportions in which the various factors are employed at the aggregate level. We shall characterize the equilibria of the model and then study how equilibrium factor usage changes in response to changes in factor prices.

The economy has two sectors and produces a consumption good and an industrial good denoted B and I respectively. There are three inputs: labour (L), capital (K) and oil (ϑ). Oil is not produced domestically, so the economy takes the price of oil p_0 as given by the monopolistic oil exporter. In order to simplify the analysis, the production functions of this country are assumed to be of the fixed proportion type

$$B = \min(L^B / a_1, \vartheta^B / b_1, K^B / c_1) \tag{1}$$

where L^B, ϑ^B and K^B denote inputs of labour, oil and capital into the production of the consumption good, and a_1, b_1 and c_1 are the technical factor-output coefficients. Similarly, the production function for the industrial good is

$$I = \min(L^I / a_2, \vartheta^I / b_2, K^I / c_2). \tag{2}$$

associated

The associated competitive price equations are then

$$p_B = a_1 w + b_1 p_0 + c_1 r p_I \qquad \text{and} \tag{3}$$

$$p_I = a_2 w + b_2 p_0 + c_2 r p_I \tag{4}$$

where w denotes wages, p_0 denotes the price of oil, r the quasi rent of capital, p_I the price of the industrial good, and p_B the price of the consumption good. $p_I r$ is then the user's cost of capital[2] which enters as a cost. Although the two production functions show fixed proportions in the use of factors, they are assumed below to have very different oil-output coefficients. We assume that factor supplies are sensitive to prices. If the price of the consumption good is the unit of measurement, then labour supply is responsive to real wages:

$$L = \alpha\left(\frac{w}{p_B}\right) \tag{5}$$

and available capital is a function of the rate of profit[3] r, i.e.

$$K = \beta r. \tag{6}$$

Next we formulate the demand behaviour, postulating that at equilibrium the value of consumption B equals wage income:

$$p_B B^D = wL. \tag{7}$$

The market equilibrium conditions are

$$K = c_1 B^S + c_2 I^S (\text{i.e. } K^S = K^D) \tag{8a}$$

$$L = a_1 B^S + a_2 I^S (\text{i.e. } L^S = L^D) \tag{8b}$$

$$B^D = B^S \tag{8c}$$

$$I^D + X = I^S \tag{8d}$$

$$p_I X = p_0 \vartheta \tag{8e}$$

where X denotes exports of I and the superscripts D and S indicate domestic demand and supply respectively. The last equation is a balance of payments condition.

At equilibrium, the national income identity (national demand equals national income) for this model is

$$p_B B^D + p_I I^D = wL + rp_I K. \tag{9}$$

To summarize, the model's exogenous variables are the technical coefficients (a_1, a_2, b_1, b_2, c_1, c_2), the parameters α and β denoting the responses of domestic factor supplies to prices, and the price of oil, p_0. The model can be formalized as a general equilibrium system given by eleven equations in twelve endogenous variables. The equations are: (1), (2), (5), (6), (7), (8a-e) and (9). The endogenous variables are: supply of I, I_S: demand for I, I^D; exports of I, X: supply of B, B^S: demand for B, B^D; rate of profit, r; price of B, p_B; price of I, p_I; wages, w; labour employed, L; oil used, ϑ; and capital used, K. The accounting identity (9) is always satisfied when all markets are in equilibrium.

As there are eleven equations and twelve unknowns, the system can be solved in the usual general equilibrium fashion by considering one good as a numeraire. The prices that emerge for the other goods are therefore relative prices. We choose B to be the numeraire, so that $p_B = 1$.

For any given price of oil and set of technological and behavioural parameters, equations (1)–(9) determine a locally unique general equilibrium of the economy. Our next step is to study how this equilibrium changes as the price of oil changes, and in particular how the amount of capital used varies across equilibria in response to changes in the price of oil.

The following is very much an exercise in computing the general equilibrium of the model. As the price of oil varies, the equilibria of this model will generally describe a one-parameter family, i.e. a curve in the space of endogenous variables. Along this curve the price of goods, wages and interest rates, total output of each good, the relative price of imports and exports (i.e. terms of trade) and the amount of exports are all endogenously related. Our next goal will be to study their relation across equilibria.

First note that, from the production functions (1) and (2), one can obtain demand equations for factors L, K and ϑ at each level of output, assuming that factors are used efficiently:

$$L^D = B^S a_1 + I^S a_2 \tag{10}$$

$$K^D = B^S c_1 + I^S c_2 \tag{11}$$

$$\vartheta^D = B^S b_1 + I^S b_2. \tag{12}$$

Equations (10) and (11) imply that, when factors are used efficiently,

$$B^S = (c_2 L - a_2 K) / D \tag{13}$$

$$I^S = (a_2 K - c_2 L) / D \tag{14}$$

where D is the determinant of the matrix $\begin{pmatrix} a_1 & a_2 \\ c_1 & c_2 \end{pmatrix}$.

The price equations (3) and (4) can be regarded as a system of two equations in two variables, w and r, when p_0 is a constant. From these equations one obtains

$$w = \frac{(p_B - b_1 p_0)c_2 - (p_I - b_2 p_0)c_1}{D} \tag{15}$$

$$r = \frac{a_1(p_I - b_2 p_0) - a_2(p_B - b_1 p_0)}{D p_I} \tag{16}$$

Substituting K and L from (5) and (6) into (13), and then w and r from (15) and (16), one obtains the equilibrium values of the supply of basic goods B as a function only of their price p_I.

$$B^S = \frac{(c_2 \alpha w - a_2 \beta r)}{D} = \tag{17}$$

$$\frac{\alpha c_2}{D^2}(c_2 + p_0 N - c_1 p_I) + \frac{\beta a_2}{D^2}\left(\frac{p_0 M}{p_I} + \frac{a_2}{p_I} - a_1\right)$$

where $M = a_1 b_2 - a_2 b_1$ and $N = c_1 b_2 - b_1 c_2$. Similarly, from (14) one obtains

$$I^S = \frac{\beta a_1}{D^2}(\alpha_1 - \frac{p_0 M}{p_I} - \frac{a_2}{p_I}) + \frac{\alpha c_1}{D^2}(p_I c_1 - c_2 - p_0 N). \tag{18}$$

Now, from the demand relation (7), $B^D = wL$. and from (9).

$$I^D = rK \tag{19}$$

at equilibrium. Therefore, from $B^S = B^D$ and $I^S = I^D + X$, one obtains the following expressions from (17) when $p_B = 1$:

$$\alpha c_2(c_2 + p_0 N - c_1 p_I) + \beta a_2\left(\frac{p_0 M}{p_I} + \frac{a_2}{p_I} a_1\right) \tag{20}$$

$$= \gamma\alpha[(1 - b_1 p_0)c_2 - (p_I - b_2 p_0)c_1]^2.$$

The implicit function theorem implies that from (20) one can obtain, at least locally, a function $p_I = p_I(p_0.)$. Therefore, since p_0 is given, an equilibrium value of p_I can be obtained. This, from (17) and (18), yields the supply of B and I, B^S and I^S, at equilibrium. From (15) and (16) one obtained wages and profits w and r, and from (5) and (6) the equilibrium use of inputs K and L. This determines I^D (see (19)), so that the volume of exports X is also known; therefore imports of oil can be computed from (8e). Thus the model is 'closed', i.e. its equilibria are determined (and locally unique) when p_0 is given. When p_0 changes, the equilibrium values of all endogenous variables will change, in particular the use of capital. Our next goal is to compute this relationship across equilibria.

We now make an assumption that simplifies the computations: we assume that $c_1 = 0$, i.e. that B requires no capital inputs. This is not strictly necessary to obtain the results: all that is required is that B be significantly less capital-intensive than I in order to obtain substitution in the aggregate use of factors. One can think of B as a non-traded, relatively labour-intensive commodity, such as services.

From (20), and using this assumption, one obtains an explicit expression for $p_I = p_I(p_0)$

$$p_I = \frac{a_2 + p_0 M}{\gamma b_1 p_0(b_1 p_0 - 1) + a_1} \tag{21}$$

where $\gamma = \alpha c_2^2 / \beta a_2$.

Consider now the possible range of variation of p_0. From the price equation (3), since $c_1 = 0$, $w \geq 0$ implies $1/b_1 \geq p_0 \geq 0$. Now, from (3)

$$w = \frac{1 - b_1 p_0}{a_1}. \tag{22}$$

Therefore (4) implies

$$r = \frac{1}{c_2} - \frac{a_2 + p_0 M}{a_1 c_2 p_I}. \tag{23}$$

Substituting for p_I from (21) we obtain

$$r = \frac{\alpha c_2 b_1}{\beta a_1 a_2}(p_0 - b_1 p_0^2). \tag{24}$$

Therefore $r = 0$ both when p_0 is zero and when p_0 assumes its maximum value $1/b_1$. The change in the rate of profit as the price of oil varies is

$$\frac{\partial r}{\partial p_0} = \frac{\alpha c_2 b_1}{\beta a_1 a_2}(1 - 2 p_0 b_1). \tag{25}$$

Since

$$\frac{\partial r}{\partial p_0} = 0 \Leftrightarrow p_0 = \frac{1}{2b_1} \tag{26}$$

and r is quadratic in p_0, it follows that the rate of profit as an increasing function of p_0 for $p_0 < 1/2b_1$, and a decreasing function for $p_0 > 1/2b_1$. Since r takes its maximum value when $p_0 = 1/2b_1$, the maximum value of r is

$$r_{max} = \frac{1 \alpha c_2}{4 \beta a_1 a_2}. \tag{27}$$

Figure 1 shows the relationship between r and the price of oil, p_0. The intiutive explanation of this relationship is straightforward. An increase in the price of oil has two opposing effects on the demand for capital – a substitution effect and an income effect. The substitution effect occurs because the relative price of the oil-intensive produced good rises, shifting demand to the capital-intensive good and thus raising the demand for and the level of use of capital. This bids up the return to capital. The income effect occurs because an increase in the price of oil reduces aggregate demand and thus the demand for factors, tending to lower the price of capital. The income effect can be shown to dominate at higher price levels (see Chichilnisky (1981)).

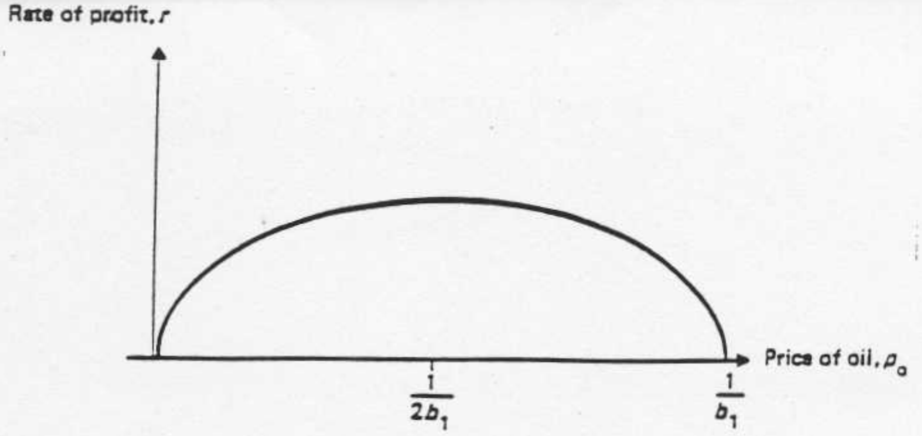

Figure 1   The rate of profit and the price of oil

With this information, we are in a position to analyse the response of capital used to changes in the price of oil. From (6), the supply of capital is just Br and from (8a) capital demanded and supplied are equal at equilibrium. Hence if K is equilibrium capital use,

$$\frac{\partial K}{\partial p_0} \gtrless 0 \text{ as } p_0 \lessgtr \frac{1}{2b_1}.$$

Thus, within this model, capital and energy are *total substitutes at low energy prices* and *total complements at high energy prices*. Whether capital and energy are total complements or substitutes is a characteristic of the equilibrium position of the model, which depends on the price of energy and the parameters of the model. For the particular model presented here, the total cross elasticity of demand actually has a very simple analytical form. When defined as

$$\xi = \frac{\partial K p_0}{\partial p_0 K}$$

where all values of variables are equilibrium values, derivatives are evaluated across equilibria. Using (6), (8a), (24) and (25), it can be shown that

$$\xi = \frac{1 - 2b_1 p_0}{1 - b_1 p_0}.$$

The total cross price elasticity is thus positive and is between one and zero for $0 \leq p_0 < 1/2b_1$, is zero for $p_0 = 1/2b_1$, and varies from zero to minus infinity as p_0 varies between $1/2b_1$ and $1/b_1$. This is summarized in Figure 2.

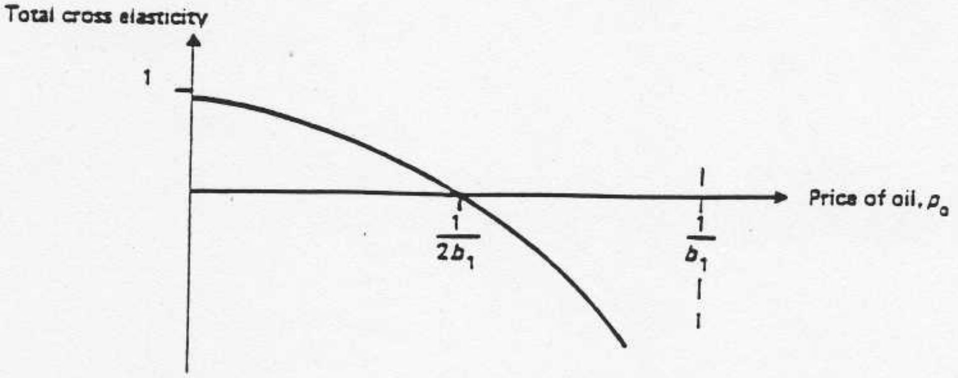

Figure 2   Behaviour of the elasticity ξ with the price of oil

Conclusions

We have introduced the concept of the *total* cross price elasticity of demand between energy and capital: it is an elasticity that takes into account the full range of adjustments that occur in a multisector economy after the price of energy changes. It is a comparative static measure, recording the change in the general equilibrium configuration of factor use consequent upon a factor price change. It incorporates the effects of potentially important sources of substitutability, such as those on the demand side that are not adequately captured by analysis of a production function, and so measures the total response from all sources to a price change. It is obviously a long-run concept, applicable on a time-scale over which all adjustments may be assumed to have been completed. It is clear that there are a number of contexts in which this total elasticity will be more relevant than any partial concept – for example, when predicting the effect on total energy consumption of a price change.

The concept of total cross rice elasticity has been illustrated by reference to a model which is simple enough to be tractable, yet highlights the main feature. It shows, for example, that capital and energy may be substituted at the aggregate level, even though in every production process they are used in fixed proportions. This illustrates clearly the importance of considering the full range of responses to a price change. For the particular model considered, capital and energy are substitutes at low energy prices and complements at high energy prices. This very simple relationship is heavily dependent on our simplifying assumption that c_1, the capital-output coefficient in the consumption goods sector, is zero. Without this simplification, the relationship between p_0 and r, and hence K, would be of the fourth order. This would lead to three regime switches between complementarity and substitutability, as opposed to the one transition of the present model. Such a possibility has to be borne in mind when comparing the predictions of the model with the stylized facts of capital-energy complementarity in the US and substitutability in Canada and Europe. It also has to be borne in mind that the price of oil, p_0, is in fact the price of oil relative to the price of consumption goods: it is not immediately clear whether this is higher or lower in Europe than in the US. Finally, it is of course, the case that existing studies have been designed to measure a different elasticity: a rather different model specification would be required to estimate the total cost price elasticity. This will be the subject of a forthcoming paper.

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Notes

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2. We have in mind an interpretation of this model as a temporary general equilibrium model. Industrial goods produced in this period may be used *inter alia* to augment the capital stock in the next period.
3. One could think of a situation where the capital stock consists of a number of machines of different ages and productivities. As the rate of profit rises, an increasing number of these will be brought into operation.