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Chaotic price dynamics, increasing returns and the Phillips curve

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Abstract

Chaotic behavior of prices can emerge as a robust result from a very simple and standard price adjustment process. Consider the dynamics of prices adjusting according to supply and demand in economies with increasing returns to scale. Increasing returns in production implies the existence of a globally attracting set of prices, containing a stable disequilibrium price, within which the motion of the system is chaotic. This property holds for any step size in the price adjustment process when consumption and leisure are complementary. We prove that long-run statistical properties of the system's behavior in this set are described by an ergodic measure. Price dynamics drive the system into the globally attracting region, and then chaotic motion takes over. On average according to this measure there is excess supply. We suggest possible empirical implications of our analysis, particularly with respect to the relationship between wages changes and the demand for labor, the "Phillips curve".

JEL classification: C62; D50; E24

Keywords: Chaos; Discontinuous; Disequilibrium; Phillips curve; Employment

1. Introduction

Chaotic behavior of prices can emerge as a robust result from a very simple and standard price adjustment process. Consider the dynamics of price adjustment in economies with increasing returns to scale, with the dynamics given by the usual adjustment according to the laws of supply and demand. Heal (1982, 1991) and

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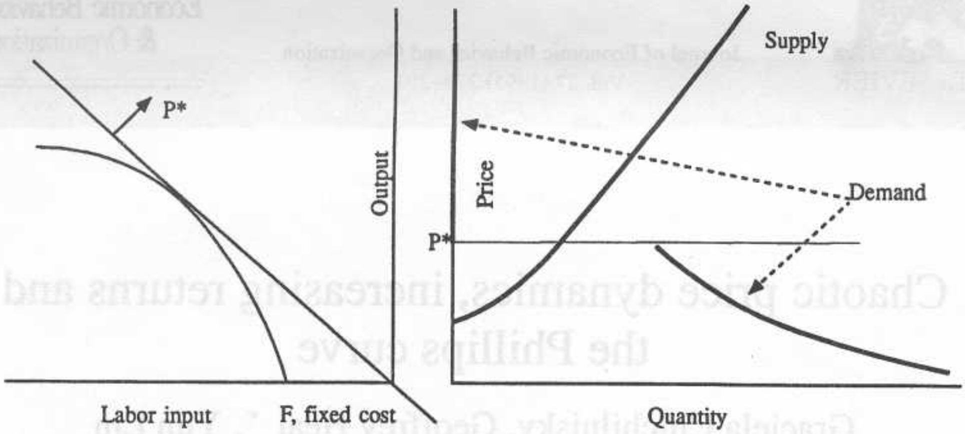

Fig. 1. A production function with a fixed cost F followed by diminishing returns, and the labor demand curve to which it gives rise. This does not intersect the supply curve, so that there is no market clearing price. P^* is a "stable disequilibrium price".

Chichilnisky and Heal (1987) show that such economies may have a "stable disequilibrium price", i.e., a price vector that is locally stable although not a market clearing price. The stable disequilibrium price, is a price vector at which the excess demand function of the economy is discontinuous. It is in fact the price vector that would clear markets and give a competitive equilibrium in the convex economy defined by replacing non-convex production sets by their convex hulls. This is illustrated in Fig. 1, in which p^* is the stable disequilibrium price. Fig. 1 shows, on the left, a non-convex production possibility set and the price vector p^* at which the firm's input demand is discontinuous, and on the right the resulting discontinuous input demand function and a continuous supply function. For $p > p^*$ there is excess supply, and vice versa. At p^* excess demand takes two values, neither of which is zero. Clearly if price falls when supply exceeds demand and vice versa, then it will tend to p^*. This is not however a market clearing price: there is in fact no such price (for a more detailed discussion, see Heal (1981)) [1].

In the Section 2 we set out the model used in the paper. Section 3 contains the main theoretical result. These are:

[1] The previous analysis showed that whenever the price vector is away from the "stable disequilibrium", it moves to reduce the distance from that point. Behavior at the stable disequilibrium was not analyzed: the behavior of the system is not defined at there. A natural intuition is that at that point, and in a neighborhood of it, there is behavior that is "unusual". The system reaches the point p^* with positive velocity from either side, so that it is natural to think of it overshooting. In the present paper we address these issues: we analyze the dynamics of a discrete Walrasian system, focusing particularly on behavior in a neighborhood of the stable disequilibrium, and establish that it is either strongly chaotic or cyclical. The intuition that prices overshoot p^* and behave irregularly in that neighborhood is justified. We show that on average over time the price exceeds p^*, so that on average there is excess supply in the economy.

1. *Theorem 1*: the existence of a globally attracting set of prices, containing the stable disequilibrium price, within which the motion of the system is chaotic. Long-run statistical properties of the system's behavior in this set are described by an ergodic measure. Walrasian price dynamics drive the system into this region, and then chaotic motion takes over. This result is true for any specification of the economy where the adjustment has step size greater than a specified minimum. In addition, for an open class of preferences and technologies showing complementarity between consumption and leisure, this is also true for any step size in the adjustment process. The average price according to the ergodic measure, is one at which supply exceeds demand, so that on average there is excess supply.
2. *Theorem 2*: when substitution in consumption is extensive, then the price dynamics converge to a period-two cycle, with the stable disequilibrium price located between the two limiting points.

Section 4 suggests an application of the earlier results to an interpretation of the Phillips curve relationship between unemployment and wage changes. We argue that if there are increasing returns in the employment of labor, then the labor market may share some of the characteristics of our model, namely the absence of a market clearing wage, and chaotic behavior of wages within a certain range. This could generate time series on wage changes and unemployment with the characteristics of the Phillips curve. The policy implications of such a relationship within our framework would be very different from those normally attributed to the Phillips relationship.

Although chaotic behavior of a price-adjustment process has been noted before, the case studied here is particularly robust, both in the sense of being characterized by an ergodic measure, and also in the sense of holding for all step sizes in the adjustment process and for an open set of parameter values. To the best of our knowledge, this is also the first time that it has been possible to characterize the sign of the average excess demand as prices switch between regimes of excess supply and excess demand: on average the chaotic behavior in our model corresponds to excess supply [2].

2. The economy

In this section we set out the model used in the paper. There is a single input, labor, and a single output, a consumption good. These are produced and consumed

[2] Another distinctive technical feature of our analysis, is that the state transition function is a discontinuous map. The technical argument builds in part on recent results due to Keener (1980) on chaotic behavior in piecewise continuous difference equations. The methodology of "chaotic systems" in economics is clearly reviewed in Day and Pianigiani (1991). For an analysis of the methodological and conceptual issues associated with this type of system, the reader is referred to Baumol and Benhabib (1989).

respectively by a single firm and consumer. The firm's technology is given by the production possibility set:

$$y = \begin{cases} 0 & \text{if } L \leq F \leq 1 \\ A(L-F)^{\alpha} & \text{otherwise} \end{cases} \tag{1}$$

where $0 < \alpha < 1$. There is a fixed cost introduced by a minimum input requirement of F: once this is met, output shows diminishing returns. F is assumed to be less than 1, which is the total labor supply.

It is easy to show that this production function will give rise to the conventional U-shaped average cost curve so widely assumed in microeconomics text books: average costs fall initially as the fixed cost is spread over an increasing output, but then increase as diminishing returns bite. It is not difficult to justify the concept of fixed cost: clearly there are large setup costs in many industries requiring either initial R&D or substantial initial investments in plant and equipment. In the present model the fixed cost is in terms of labor. One interpretation is to think of the fixed labor costs as a metaphor for more general fixed costs: we are keeping the dimensionality down to two (one input, one output) to make the dynamical system tractable. Another interpretation is to think of the administrative, accounting, organizational and sales staff required to run a corporation, all of whom constitute a fixed labor requirement which is independent of the level of output, at least for a range of output levels. An alternative (and rather ingenious [3]) illustration of a fixed minimum labor requirement is a "bucket chain" passing water from a lake to a fire: until there are enough people to form the chain, no water will be passed.

The price of the output is normalized to be one, and W is the wage rate. Hence profits π are given by

$$\pi = A(L-F)^{\alpha} - WL, \quad L \geq F. \tag{2}$$

The first order conditions for profit maximization define the demand for labor, (see equation 3), which is discontinuous, as shown in Fig. 1.

$$L_D = \begin{cases} \left(\dfrac{W}{\alpha A}\right)^{\frac{1}{\alpha-1}} + F & \text{if } W \leq W^* \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The wage rate at which the demand for labor is discontinuous is found by substituting the labor demand function for $W \leq W^*$ into (2) and equating profit π to zero, which gives

$$W^* = A\alpha^{\alpha}\left[\frac{(1-\alpha)}{F}\right]^{(1-\alpha)}. \tag{4}$$

[3] Due to a referee.

Labor demand at W^* equals $F/1-\alpha$, which is independent of the scale parameter A.

Preferences are given by the CES utility function:

$$u(Y,L) = \left[Y^{\beta} + (1-L)^{\beta}\right]^{\frac{1}{\beta}},\ \beta \in (-\infty,1)$$

where maximum labor supply is scaled to be one and Y is the worker's consumption level. The consumer maximizes utility, which gives the labor supply function

$$L_S = \frac{1}{1+W^{\frac{\beta}{\beta-1}}},\ \beta \in (-\infty,1). \tag{5}$$

If β is a large negative number, so that consumption and leisure are consumed in approximately fixed proportions[4], the labor supply curve is backward-bending: for β near unity, giving a high level of substitutability between consumption and labor, the supply curve has a positive slope. Hence the excess demand function for labor is

$$Z(W) = \begin{cases} \left(\frac{W}{\alpha A}\right)^{\frac{1}{\alpha-1}} + F - \frac{1}{1+W^{\frac{\beta}{\beta-1}}} & \text{if } W \le W^* \\ -\frac{1}{1+W^{\frac{\beta}{\beta-1}}} & \text{if } W > W^* \end{cases} \tag{6}$$

3. Dynamic behavior

In this section we establish the main results. Proposition 1 confirms the existence in our model of a stable disequilibrium price, while Theorems 1 and 2 establish respectively the conditions for chaotic and cyclical behavior.

Price dynamics are governed by the following equation:

$$W_{t+1} = W_t + \lambda Z(W_t), \tag{7}$$

where $\lambda > 0$ is given. Price adjusts proportional to the excess demand of the current period. For convenience, define the map:

$$W_{t+1} = \theta(W_t; A, \alpha, \beta). \tag{8}$$

[4] Consumption and leisure are consumed in approximately fixed proportions in any recreational activity which requires consumer goods as an input.

Proposition 1. *For all fixed costs greater than a minimum $F^* \equiv F^*(A,\alpha,\beta) \leq 1 - \alpha$, there is a stable disequilibrium price and no Walrasian equilibrium exists.*

Proof. From equation (3) we know that labor the demand curve has two segments. The segment for $W < W^*$ has as its boundary $L^*(W^*) = F/1-\alpha$ and shifts to the right as F increases (from (3)): W^* decreases as F increases (from (4)). Choose F^* to be the largest F such that the labor demand curve for $W < W^*$ and the labor supply curve intersect. Since labor supply is bounded above by 1, we have $F^* \leq 1 - \alpha$. It is therefore clear that the labor demand curve for $W < W^*$ will not cross the supply curve if $F \geq F^*$. For $W \geq W^*$, labor demand is constant and equal to zero. Labor supply converges to zero only if W goes to infinity and $\beta < 0$.

The assumption $F^* < F \leq 1 - \alpha$ will be maintained throughout our discussion. For $F < F^*$, there always exists a stable Walrasian equilibrium. The dynamic adjustment processes for that case will not be discussed here. If $F > 1 - \alpha$, then the labor demand is either equal to zero or greater than the maximum labor supply.

We split our discussion into two parts. In the first part (Theorem 1), we look at the case where preferences display complementarity between leisure and consumption ($\beta < 0$) and find that price dynamics demonstrate chaotic behavior which persists for all step sizes, and in particular as λ decreases to zero. In the second part (Theorem 2), we look at the case where consumption and leisure are substitutes ($0 < \beta < 1$). Chaotic behavior may also be found: however, it disappears as λ becomes smaller than some critical value λ^*, and is replaced by periodic behavior. Since chaotic behavior would not occur for small λ without fixed costs (i.e., with $F = 0$, in which case we have a convex economy: see the arguments in Day and Pianigiani (1991), section 2), we may conclude that the fixed cost is responsible for the chaotic behavior in the first case ($\beta \leq 0$).

We now establish the main result of the paper, Theorem 1, which shows that for preferences displaying complementarity between leisure and consumption, the discrete Walrasian adjustment process (7) leads to chaotic behavior with an associated ergodic measure for any value of the adjustment parameter λ. Theorem 2 deals with the case of substitutability between consumption and leisure, and establishes the existence of a limit cycle. From now on, we assume that the fixed cost F is sufficient to ensure the existence of a stable disequilibrium price, i.e., $F^* < F$, and in addition that $F < 1 - \alpha$, so that the fixed cost is bounded below the total labor supply by an amount depending on the nature of returns to scale after the fixed cost is met.

Theorem 1. *Consider any adjustment size λ and $\alpha \in (0,1)$. Assume that consumption and leisure are complements so that the labor supply curve is backward sloping, i.e., $\beta \leq 0$. Then for α sufficiently close to one, i.e., the production function close to linear after the fixed cost is met:*

(i) There exists upper and lower "trapping values" of the real wage W, $\underline{W}$ *and* $\overline{W}$ *and a time* $T > 0$*, such that for any initial value of the real wage* W_0 *and all times* $t > T$*, the real wage is in the "trapping set", i.e.,* $W_t \in [\underline{W}, \overline{W}]$*.*

(ii) Within the interval $[\underline{W}, \overline{W}]$ *the behavior of (7) is chaotic in the sense that there exists a unique invariant measure* μ *on* $[\underline{W}, \overline{W}]$ *that is absolutely continuous with respect to the Lebesgue measure with the following property: for almost any initial conditions and any measurable subset S of* $[\underline{W}, \overline{W}]$*,* $\mu(S)$ *is the average fraction of the total number of periods that a trajectory spends in S.*

Proof. A crucial step is to establish that $W_{t+1} = \theta(W_t)$ or $W_{t+i} = \theta^i(W_t)$, for some integer i, is an expansive map (Day and Pianigiani (1991) p. 45, Theorem 3). A map is expansive if the absolute value of its derivative is bounded above unity, Lebesgue almost everywhere. Since the proof for $\beta = 0$ is slightly different from that of $\beta \in (-\infty, 0)$, we look at the two cases separately.

Case 1. $(-\infty, 0)$*. (see Fig. 2a)*

We show that (8) is an expansive map. For $W \geq W^*$,

$$\frac{\mathrm{d}W_{t+1}}{\mathrm{d}W_t} = 1 + \lambda \frac{\beta}{\beta - 1} \frac{1}{\left(1 + W^{\frac{\beta}{\beta-1}}\right)^2} W^{\frac{1}{\beta-1}}. \tag{9}$$

In this case, $\mathrm{d}W_{t+1}/\mathrm{d}W_t$ is clearly greater than 1.

For $W < W^*$,

$$\frac{\mathrm{d}W_{t+1}}{\mathrm{d}W_t} = 1 + \lambda \frac{1}{\alpha - 1} \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha-1}} (W_t)^{\frac{2-\alpha}{\alpha-1}} + \lambda \frac{\beta}{\beta - 1} \frac{1}{\left(1 + W^{\frac{\beta}{\beta-1}}\right)^2} W^{\frac{1}{\beta-1}}. \tag{10}$$

With some manipulation we get

$$\frac{\mathrm{d}W_{t+1}}{\mathrm{d}W_t} = 1 + \frac{\lambda}{W}\left[\frac{\beta}{\beta-1} L_S(W) - \frac{\beta}{\beta-1}[L_S(W)]^2 - \frac{1}{1-\alpha}(L_D(W) - F)\right].$$

We need to show that the right-hand side is less than -1 or to show

$$\frac{\lambda}{W}\left[\frac{\beta}{\beta-1} L_S(W) - \frac{\beta}{\beta-1}[L_S(W)]^2 - \frac{1}{1-\alpha}(L_D(W) - F)\right] < -2.$$

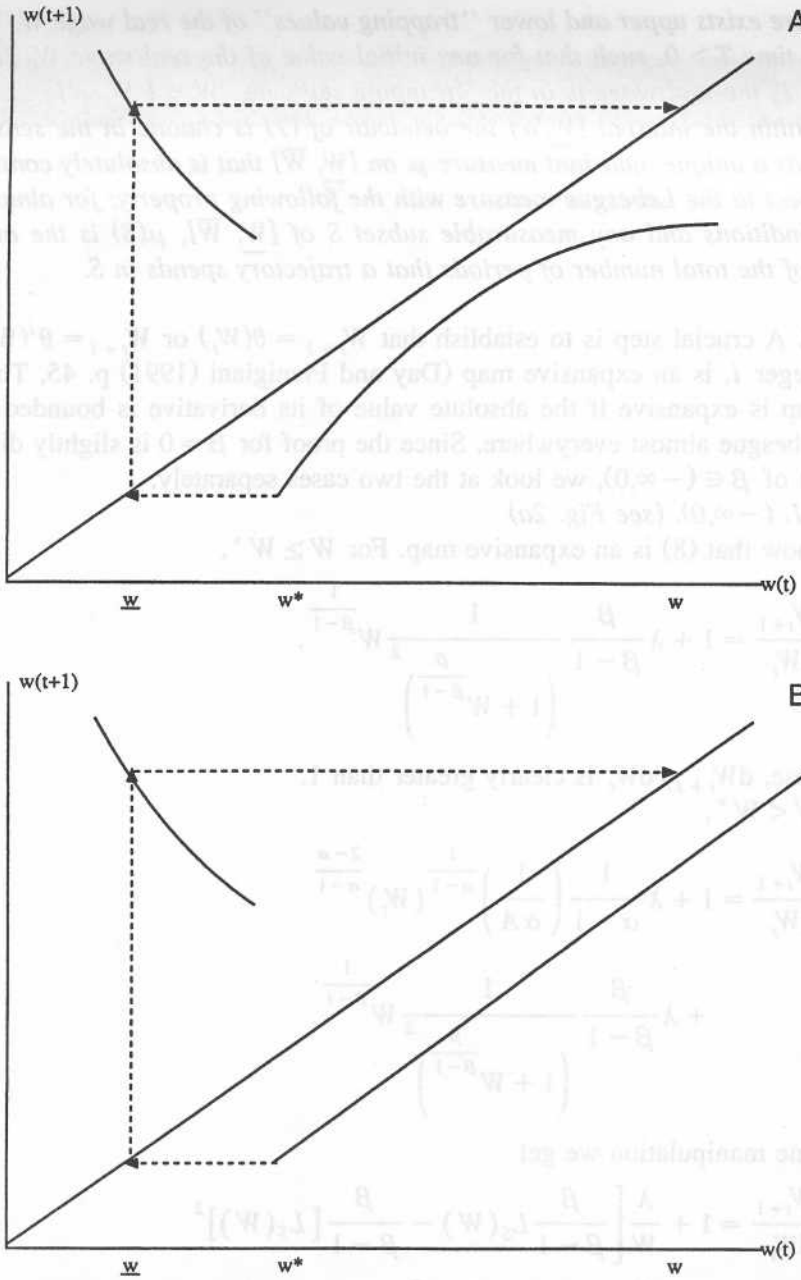

Fig. 2. (a) Transition map in the case when β is negative. (b) Transition map in the case when β is zero.

A sufficient condition would be

$$\frac{F}{1-\alpha}\left[\frac{\alpha}{1-\alpha}-\frac{1}{4}\frac{\beta}{\beta-1}\right]>\frac{2A}{\lambda}. \tag{11}$$

We used several inequalities

$$W < A,\ L_S(W) - (L_S(W))^2 < \tfrac{1}{4},\ L_D(W) - F > \frac{\alpha}{1-\alpha}F,$$

which can be derived easily. Inequality (11) will always hold if α is sufficiently close to unity, i.e., returns to scale are close to constant once the fixed costs are met. An application of Day and Pianigiani's theorem 3 now proves the theorem for $\beta \in (-\infty,0)$.

Case 2. $\beta = 0$. (see Fig. 2b) In this case, labor supply equals 1/2 for all W and $\mathrm{d}W_{t+1}/\mathrm{d}W_t = 1$ for $W \geq W^*$. Theorem 3 of Day and Pianigiani does not apply. Instead, Corollaries 2 and 3 to their theorem 4 (Day and Pianigiani (1991), p. 47) will be used. Basically, we need to find an integer i, such that the map $W_{t+i} = \theta_i(W_t; A, \alpha, \beta)$ is expansive.

Define $\underline{W} = W^* - 0.5\lambda$, and $\overline{W} = \underline{W} + \lambda Z(\underline{W})$. These two values form the interval $[\underline{W}, \overline{W}]$ of point (i) of the Theorem. Notice for any W_t located in the range $[W^*, \overline{W}]$, the price adjustment will follow $W_{t+1} = W_t - 0.5\lambda$, and after some finite number of periods, it will drop to the range $[\underline{W}, W^*]$. It is easy to see that for all initial prices located in $[W^*, \overline{W}]$, $\overline{W}$ is the initial value from which the system will take the longest number of periods, say k, to reach a point lower than W^*. Consider the map

$$W_{t+k} = \theta^k(W_t). \tag{12}$$

Differentiate with respect to W_t and by the chain rule, we have

$$\frac{\mathrm{d}W_{t+k}}{\mathrm{d}W_t} = \frac{\mathrm{d}W_{t+k}}{\mathrm{d}W_{t+k-1}} \frac{\mathrm{d}W_{t+k-1}}{\mathrm{d}W_{t+k-2}} \cdots \frac{\mathrm{d}W_{t+1}}{\mathrm{d}W_t}. \tag{13}$$

For any initial $W_t \in [\underline{W}, \overline{W}]$, and its generated sequence $\{W_t, W_{t+1}, \ldots, W_{t+k}\}$, there exists at least one W_{t+i} which belongs to $[\underline{W}, W^*]$, where the derivative is less than -1 under our assumptions. So at least one of the terms on the right-hand side of equation (11) is less than -1, all the other terms are either one, if $W \in [W^*, \overline{W}]$, or less than -1, if $W \in [\underline{W}, W^*]$. Their product in absolute value must be greater than 1. So the map $W_{t+k} = \theta^k(W_t)$ is expansive. Theorem 1 now follows from corollaries 2 and 3 of Day and Pianigiani, p. 47. This completes the proof of Theorem 1.

Finally we characterize the behavior of equation (7) describing the price dynamics for the case when $0 < \beta < 1$, i.e., consumption and leisure are substitutes. Recall that Theorem 1 addressed the case of $\beta < 0$, and established that for any adjustment parameter λ chaotic behavior is possible. With $0 < \beta < 1$ chaotic behavior is still possible, but only for λ in excess of a lower bound. In this case, for small enough values of the adjustment parameters λ, the system has a two-period orbit which is both structurally and dynamically stable. Formally,

Theorem 2. *If consumption and leisure are substitutes and the supply curve for labor slopes forward, i.e.,* $\beta \in (0,1)$*, then there exists a critical adjustment parameter* $\lambda^* \equiv \lambda^*(A,\alpha,\beta)$*, such that for adjustment parameters less than this, i.e.,* $0 < \lambda < \lambda^*$*, there is a unique, globally attracting period-two solution* $\{W_1, W_2\}$ *to the price adjustment process* $W_{t+1} = \theta(W_t; A, \alpha, \beta)$ *such that* $W_1 < W^* < W_2$*, where* W^* *is the stable disequilibrium price. Furthermore, there is a period two solution for an open set of parameter values.*

Proof. This theorem follows from discussions in section 3 of Keener (1980). We need, however, to reformulate our problem so that his results can be used.

As in the case for $\beta \leq 0$, we can define a trapping region $[\underline{W}, \overline{W}]$. $\underline{W}$ will depend on λ. Let λ_1 be the biggest value, such that $\underline{W} > 0$ for all $\lambda < \lambda_1$. Now look at equation (7) and (8). For $\alpha \in (0,1)$ and $\beta \in (0,1)$, and for both equations, the derivative dW_{t+1}/dW_t is uniformly bounded above by one. Since the derivative decreases monotonically with λ for each case, there exists a maximum λ^* which is less than or equal to λ_1, such that for $\lambda < \lambda^*$, the right-hand sides of equation (7) and equation (8) are both uniformly bounded below by zero for their corresponding domains of W. The following lemma follows from the definitions of F^* and λ^*.

Lemma 1. *For any structural parameters A,* α *and* β*, there exists* F^* *and* λ^**, such that if* $F > F^*$ *and* $0 < \lambda < \lambda^*$*, then*

1. $\theta(W; A, \alpha, \beta)$ *maps from* $[\underline{W}, \overline{W}]$ *to itself.*
2. $0 < dW_{t+1}/dW_t < 1$ *for* $\overline{W} \in [\underline{W}, W^*)$ *and* $W \in (W^*, \overline{W}]$*,*
3. $\theta(\underline{W}) > W^*$ *and* $\theta(\overline{W}) < W^*$*.*

Here (3) is a direct result of (2).

Lemma 2. *Suppose (3) of lemma 1 is true, then* $\theta(W)$ *has a period-two solution (Keener, Lemma 3.2).*

Lemma 3. *With the assumption of lemma 1, the period-two solution in the above lemma is unique, globally attracting and structurally stable* [5]*. (This is a combination of Keener's lemma 3.1, corollaries 3.16 and 3.17).*

The above discussion together with lemmas 1–3 proves Theorem 2.

The following immediate corollary describes a basic property of a time series of excess demand for labor and changes in the real wage. It observes that if we plot these against each other, they will lie in the second and fourth quadrants and demonstrate a persistent negative correlation.

[5] In the sense of holding on an open set of parameter values.

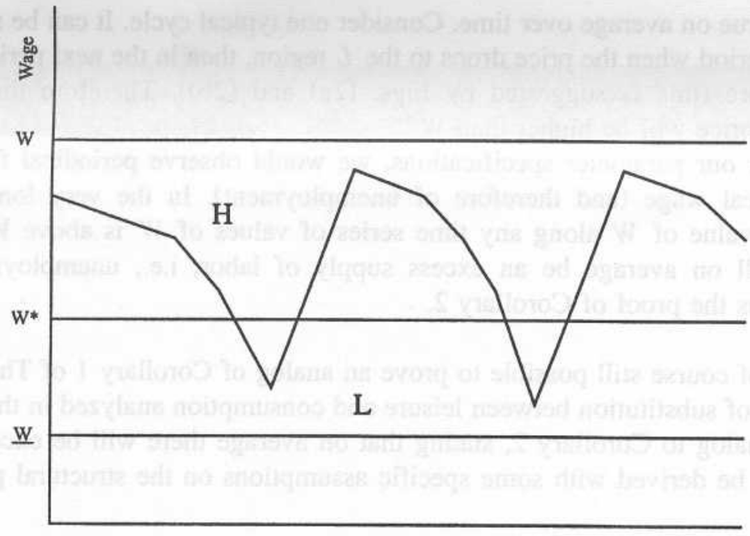

Fig. 3. An illustration of Corollary 2.

Corollary 1. *Let ΔW_t, $Z(W_t)$, $t = 1,...,\infty$ be a sequence of wage change and excess demand pairs on a trajectory of the price adjustment process (7). Then $sign(\Delta W_t) = signZ(W_t)$ for all t. Furthermore, the sequence $\{|\Delta W_t|\}$ remains bounded away from zero.*

Proof. The proof of this result is immediate from Theorem 1.

The following result characterizes the average or long-run relationship between supply and demand during the chaotic behavior of the system. It shows that on average over time there is an excess supply of labor, because on average the wage exceeds the stable disequilibrium wage.

Corollary 2. *The long-run average wage is always greater than the stable disequilibrium price, W^*, under the conditions of Theorem 1.*

Proof. Denote $[\underline{W}, W^*]$ region L and $[W^*, \overline{W}]$ region H (see Fig. 3). Consider first region L. $Z'(\overline{W})$ is less than -1 for α close to unity. So for any $W_s \in L$, $W_{s+1} \geq W^* + Z(W^*)$, which implies that W would not spend more than one period in region L.

Now suppose $W_t \in H$ in period t. Since excess demand is negative, W_{t+1} will be lower than W_t, and after some finite number of periods, say k, it must drop down to region L. Price adjustment thus displays a cyclical pattern. During each cycle, the price will stay in region L only once and in region H at least once. If we can show the average price for each cycle is greater than W^*, then this must also be true on average over time. Consider one typical cycle. It can be shown that in any period when the price drops to the L region, then in the next period it gains even more (this is suggested by Figs. (2a) and (2b)). Therefore the long-run average price will be higher than W^*.

Under our parameter specifications, we would observe periodical fluctuations of the real wage (and therefore of unemployment). In the very long run, the average value of W along any time series of values of W is above W^*. Hence there will on average be an excess supply of labor, i.e., unemployment. This completes the proof of Corollary 2.

It is of course still possible to prove an analog of Corollary 1 of Theorem 1 in the case of substitution between leisure and consumption analyzed in the Theorem 2. An analog to Corollary 2, stating that on average there will be excess supply, can also be derived with some specific assumptions on the structural parameters.

4. Persistent disequilibrium and the Phillips curve

Theorem 1 and Corollary 1 are suggestive of a novel interpretation of the statistical relationships that have often been noted between wage changes and unemployment. Consider a plot of wage changes against the associated levels of unemployment (the negative of excess demand). Theorem 1 implies that wage changes and unemployment always have the opposite sign, so that there is a negative association between wage changes (inflation) and unemployment. Furthermore, this relationship is persistent in the sense that wage changes do not go to zero over time: this is Corollary 1.

The economic implications are that within the attracting set of prices, there is always either excess supply or excess demand, which is accompanied by price changes. If we take the input to be labor and the price to be the real wage, then the price dynamics generate a time series of real wage changes and levels of unemployment: this time series will have the statistical properties of a Phillips curve (Phillips, 1958, Sargan, 1980). Theorem 1 implies that wage changes and unemployment always have the opposite sign, so that there is a negative association between wage changes (inflation) and unemployment. Furthermore, this relationship is persistent in the sense that wage changes do not go to zero over time. For a price adjustment process which converges to an equilibrium, the relationship would not persist indefinitely, but would be a transient or disequilibrium phenomenon: here it is a long-run equilibrium relationship.

In this framework, it is clear that a persistent negative relationship between wage changes and unemployment does not represent a locus of alternative equilibrium configurations. These are not alternative configurations between which a policy-maker can choose. They represent rather a stable limiting distribution of excess demand-price change pairs according to the ergodic measure of Theorem 1.

The "Phillips curve" relationship therefore has no policy implication about a trade-off between inflation and unemployment in this context: it is a by-product of price dynamics in a non-convex economy. We are able to predict from the parameters of the model whether there will be on average excess demand or excess supply in the very long run, as the system evolves within the attracting set and displays "Phillips curve-like" behavior. In a statistical sense, the economy will display chronic excess demand for or supply of labor, depending on the nature of technologies and preferences.

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